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A MECHANISM FOR RESTORING DISSIPATED ENERGY
TO A DAMPED ROTATIONAL OSCILLATORY SYSTEM

A THESIS

Presented to

The Faculty of the Division of Graduate

Studies and Research

by

William Albert Horn, Jr.

In Partial Fulfillment

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Master of Science in Mechanical Engineering

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A MECHANISM FOR RESTORING DISSIPATED ENERGY

TO A DAMPED ROTATIONAL OSCILLATORY SYSTEM

Approved:

David M. Sanborn, Chairman

Harold L. Johnson

David J. McGill

Date approved by Chairman: Nov. 15, 1972

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TABLE OF CONTENTS

TABLE OF CONTENTS (Concluded)

LIST OF TABLES

LIST OF ILLUSTRATIONS

LIST OF ILLUSTRATIONS (Continued)

LIST OF ILLUSTRATIONS (Concluded)

NOMENCLATURE

c	equivalent viscous damping coefficient, in-lbf-sec/rad, and specific heat, Btu/lbm-^{o}F
c_c	critical damping coefficient, in-lbf-sec/rad
h	heat transfer coefficient, Btu/sec-ft^2-^{o}F, and Newton-Raphson iteration factor, and numerical integration increment
k	torsional spring constant, in-lbf/rad, and Newton-Raphson iteration factor
s	Laplace transform variable
t	time, seconds, and time of single clutching operation, seconds
t_{accel}	time interval for motor acceleration, sec
t_{av}	average clutch temperature, ^{o}F
t_{trans}	time interval for clutch engagement, sec
t_1	time corresponding to clutch engagement, sec, and ambient temperature, ^{o}F
t_2	time corresponding to clutch disengagement, sec
t^*	translated time for stage II, sec
t^{**}	translated time for stage III, sec
A	exposed clutch surface, ft^2
C	heat transfer coefficient, Btu/sec-ft^2-^{o}F
F_μ	Coulomb friction coefficient for bearings, in-lbf
H	heat generated each clutch engagement, Btu
J_F	total drive inertia, in-lbf-sec^2
J_T	total load inertia, in-lbf-sec^2

NOMENCLATURE (Continued)

L[ ]	Laplace operator
N	normal force, lbf, and number of clutch cycles per hour
Q	heat generated each clutch engagement, Btu
R	friction force, lbf, and gear reduction ratio
T_f	ramp torque slope (disengagement), in-lbf/sec
TQ	torque intercept of motor torque versus speed curve, in-lbf.
Tr	ramp torque slope (engagement), in-lbf sec
T(t)	torque transmitted through clutch, in-lbf
T_O	magnitude of step torque, in-lbf, and ambient temperature, $^{\circ}F$
$T(\dot{\varphi})$	motor torque, in-lbf
T_{∞}''	ultimate clutch temperature, $^{\circ}F$
V	velocity, in/sec
W	work, in-lbf, and mass of clutch parts, lbm
α	slope of motor torque versus speed curve, in-lbf-sec/rad
μ	kinetic friction coefficient
ω_d	damped natural frequency, rad/sec
ω_n	natural frequency, rad/sec
ω_1	motor speed at time equal t_1, rad/sec
ω_2	motor speed at time equal t_2, rad/sec
φ	motor shaft angular displacement, rad
τ	specified cycle period, sec

NOMENCLATURE (Concluded)

τ_{calc}	calculated cycle period, sec
θ	tub angular displacement, rad
Θ	initial angular displacement, rad
ζ	damping ratio

SUMMARY

This thesis concerns a machine design problem which evolved from research conducted by the Whirlpool Corporation. An active area of development in the field of commercial washing machines is that of increased load capability. In vertical axis machines, however, an upper limit on load size for the conventional design (i.e., central agitator in a fixed tub) has been reached due to inadequate cleansing action. One particular approach to this subject as initiated by the above researchers is to agitate the entire tub assembly. Due to the high inertia and extreme fluid damping of this load, an efficient means of producing the desired oscillation is required.

The object of the following research was to investigate the feasibility of a new transmission to accomplish the desired tub motion. The fundamental principle underlying the proposed system is the energy storage capability of certain mechanical elements. The mechanism utilizes a torsional spring and a flywheel to store potential and kinetic energy respectively, and thus distribute the motor workload evenly throughout a cycle of oscillation. Essentially, the mechanism consists of a torsion-spring oscillator (the tub system) which receives energy from a flywheel-motor combination periodically via a clutch arrangement.

The research was entirely analytical in nature and was based on a mathematical model of the proposed system. The equations of motion were solved using Laplace transforms and the parameters which

determine the proper matching of the various equations were found by techniques of numerical analysis.

The system is demonstrated to be successful in reducing the external power requirement by allowing a smaller motor to run smoothly in one direction with little speed fluctuation. This is as opposed to any direct-drive method wherein a large motor is required to start-stop-reverse cycle repeatedly. The results primarily reflect a concern to establish the ranges of successful values for the various parameter variables involved, rather than trying to find a single optimum design.

CHAPTER I

INTRODUCTION

Background and History of the Problem

An important area of development in commercial washing machines is that of increased load capability. Associated with this goal is the requirement that the cleansing action be equal to present day standards and that complexity and power requirements do not exceed values compatible with the increased performance. If the discussion is restricted to vertical axis units, recent attempts at this goal generally fall into one of two categories.

First, there are scaled-up versions of present machines, that is, the same design but with a larger tub and central agitator. The disadvantage in this approach is the negligible washing action which is transmitted to clothes at the periphery of the tub. If the agitator blade size is increased, or if its rate of agitation is increased, the above problem is reduced, but only at the expense of increased wear on the fabrics. So, in general, this approach has reached an upper limit.

A second scheme has been investigated at the Whirlpool Advanced Engineering Laboratories in St. Joseph, Michigan. This method involves agitating the entire tub assembly. This approach provides good cleansing action for a wide range of loading due to the large area of moving surface which transmits motion throughout the tub volume. An added advantage is in reduced fabric wear due to replacing the former blades with smooth tub walls as the elements which induce fluid motion. However, in this form, the approach is unsuccessful due to the extreme power needed to oscillate the whole tub.

The problem, as approached by the above group, has been largely reduced by attacking two particular sub-problems. First, the tub inertia was reduced markedly by oscillating a perforated, lightweight basket within a larger, fixed tub which contains the water. Secondly, a torsional spring was introduced to reverse the motion at zero velocity points and thus aid the motor during the initial acceleration from rest at each end of the swing. Thus, energy storage in the spring is utilized to smooth the power demand on the motor, which is operating in its least efficient configuration, i.e., start-stop-reverse cycling. At present, however, the increased cost and complexity of these additions still outweigh the advantages.

The problem essentially requires overcoming considerable damping due to fluid sloshing, so that a high inertia load may be oscillated at a prescribed rate and amplitude. The object of this thesis is to demonstrate the feasibility of a new transmission to accomplish the desired tub motion, and to provide evidence for the justification of further development of the system.

The Proposed System

As initiated by the above research, the proposed mechanism utilizes a combination of energy storage concepts. Common mechanical devices such as a spring and a flywheel are incorporated to evenly distribute the motor workload over a cycle. The resulting motion reflects a complex interplay of energy between the spring, flywheel and the motor.

The mechanism is essentially a torsional spring, damped oscillatory system which periodically receives energy from a flywheel via a clutch arrangement. The motor is then not required to stop and start cyclically and operate in its poorest efficiency range. The possible advantage of this method lies in allowing a smaller motor to run smoothly in one direction with little speed fluctuation, thus reducing energy loss in the motor.

The expected sequence of operation is outlined as follows. If the damped oscillator (representing the tub with fluid slosh damping, and torsional spring) is released from some initial configuration other than the static equilibrium position, it will typically oscillate about the equilibrium position in a sinusoidal fashion, with successive amplitudes being diminished in an exponential decay. If now at some appropriate point, an external torque is applied of just sufficient magnitude and duration to allow the system to precisely regain its initial configuration, a continuous motion is assured. It is the purpose of the clutch to periodically connect the flywheel motor system to the oscillating load in such a way as to supply this torque.

Method of Attack

The research was entirely analytical in nature and was based on a mathematical model of the proposed mechanism. The work basically involves principles of machine dynamics, vibrations, and mathematical numerical analysis. The equations of motion were solved by the technique of Laplace transforms, and the parameter variables which determine the proper matching of these equations were found via numerical techniques for nonlinear algebraic equations. The UNIVAC 1108 digital computer was the basic tool utilized in the latter effort. The computer program is included as a design tool for any future work. As with any complex system, there is no single best design solution, so the emphasis was placed on finding a set of solution parameters which perform satisfactorily in all respects. The "optimum" design is left as the desired result to be obtained from experimental work with a prototype machine. It is intended that this thesis be the basic tool for such continued research.

CHAPTER II

MATHEMATICAL MODELING

Total System

The system to be mathematically modeled may be represented pictorially as in Figure 1 below.

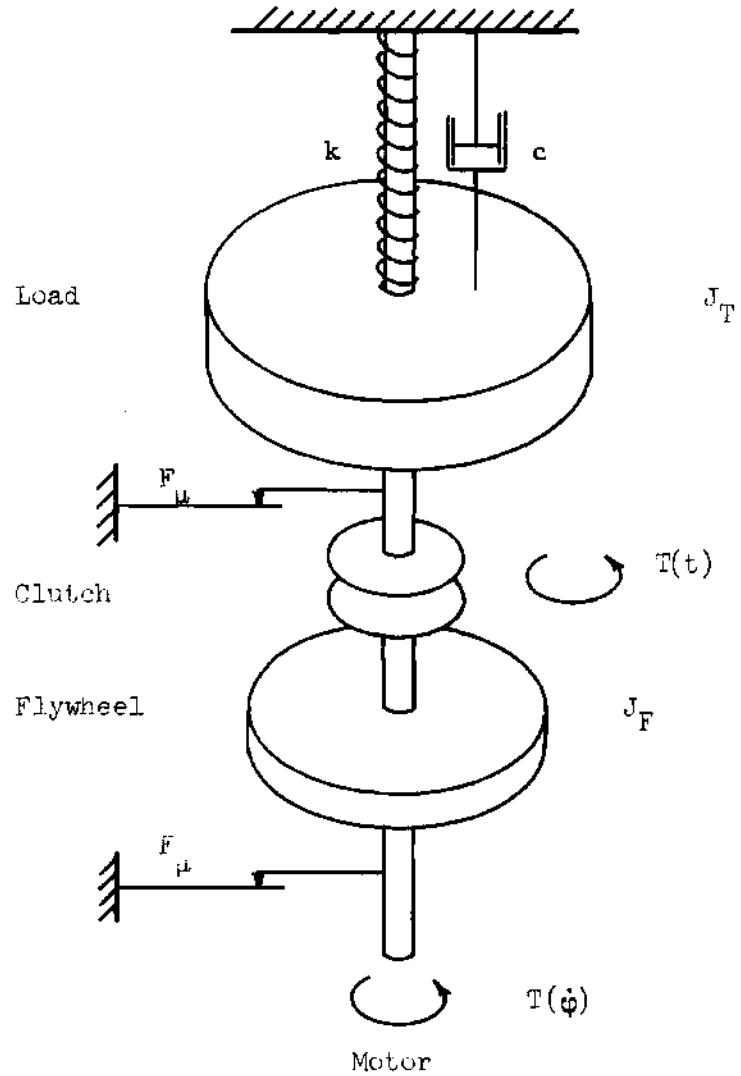


Figure 1. Schematic of Total System.

The following nomenclature is introduced:

k = Torsional spring constant

c = Load viscous damping coefficient

J_T = Total load inertia (includes shaft and load side of clutch)

J_F = Total drive inertia (includes shaft, gears, motor armature, and drive side of clutch)

$T(t)$ = Torque transmitted through clutch

$T(\dot{\varphi})$ = Motor torque

F_μ = Coulomb friction coefficient

Certainly Coulomb friction is present in bearings, gearing, etc. along with structural (hysteresis) damping, but in light of the large magnitude of the damping due to fluid sloshing at the load, these factors were regarded as being negligible at the outset. Also, an equivalent linear viscous damping coefficient was assigned as an approximation to the sloshing effect. These approximations have the advantage of making the mathematics less complex, but at the same time allow for reasonably accurate correlation with the physical system.

One of the properties of a mechanical contact clutch is the independence of transmitted torque and relative disc velocity. The basic law of friction governing this type of device is best illustrated with the familiar block sliding on a plane, Figure 2.

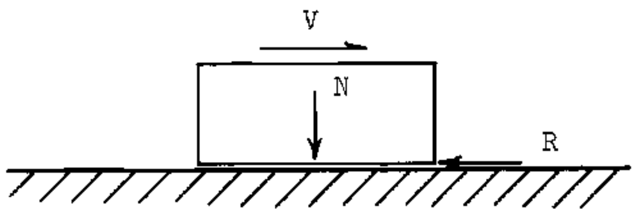


Figure 2. Coulomb Friction.

Once relative motion is established, (static friction behavior need not be considered since the clutch plates have relative motion even as they come in contact) this law states that

$$R = \mu N$$

where

R is the Coulomb friction force resisting motion,

N is the force normal to the plane of contact, and

μ is the friction coefficient (kinetic).

Therefore, R is proportional to the normal force only and is independent of relative velocity and contact area. The torsional analogy, then, has transmitted torque T independent of input and output relative speeds. Thus

$$T = T(N) = T(t)$$

since the normal force builds up according to some prescribed function of time, whether actuation is achieved electrically, mechanically or pneumatically.

It is interesting to note that another facet of clutch operation (energy loss in the form of heat) is a strong function of this relative slip. However, these subjects will be pursued further in later sections.

Mathematically, the important point is the independence of $T(t)$ on input and output speeds, for this allows the two systems representing the load portion and the drive portion to be uncoupled mathematically and examined separately.

It is worthwhile to emphasize that this mathematical uncoupling aspect of friction clutches is not necessarily a feature of other clutching devices. The common viscous clutch, for instance, has transmitted torque as a strong function of both slip velocity and viscosity. In fact, some care must be exercised in separating entirely even the system to be used with a friction clutch. The physics of force transmission by friction require that the motor side of the clutch always rotate faster than the load side for there to be any torque transmitted at all.

The following section examines clutch behavior and seeks an expression for the transmitted torque $T(t)$ to be used in the uncoupled systems called the Load Side and the Drive Side.

Mechanical Contact Clutch Characteristics

An engagement-disengagement cycle for a typical axial, disc type friction clutch (1) is shown below in Figure 3.

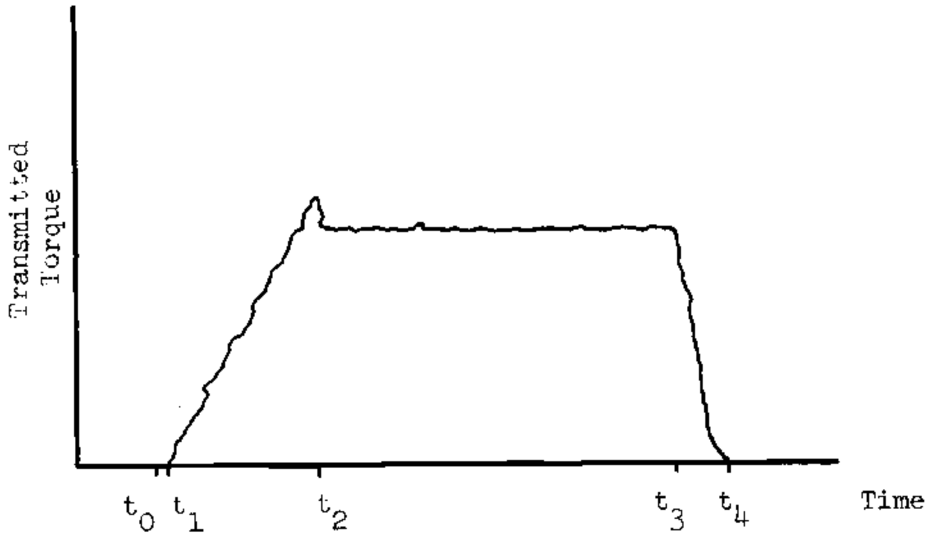


Figure 3. Torque History for a Typical Clutch.

There are three frequently used methods of actuating a contact clutch (mechanical, electromagnetic, pneumatic) each of which has a torque history similar to the plot shown above. When the clutch is actuated, time t_0, there follows a brief period of zero torque. Disc clutches have a finite gap between the plates during disengagement and it takes a finite (though small) time interval to traverse this gap. At $t = t_1$, the discs make contact. During $t_1 < t < t_2$ torque builds up to its maximum. According to Anderson (1), for most disc clutches the torque build-up is nearly linear if the applied torque is within the clutch rating. This linear build-up corresponds to a linear increase in actuating force.

This torque build-up will continue until one of two conditions is met. The speed of the load may come up to motor speed, at which time slipping ceases and the clutch plates rotate together. Or, the torque may build up to the clutch rating, i.e., maximum force on the plates, and continue to slip while transmitting the rated maximum torque. Either of these conditions continues until time t_3, at which the cycle is completed by a smooth torque drop off.

Certain irregularities and deviations exist from this theoretical straight line behavior due to wear deposits, local yielding and non-uniform pressure but the linear assumption is accurate enough for the case under consideration.

This model indicates that the transmitted torque, $T(t)$, has both ramp and step characteristics. The corresponding mathematical model is constructed using unit step functions in the following manner. If actuation is assumed to take place at $t = 0$, and the short lag interval is neglected, the initial ramp is given by

$$T(t) = u(t-0)Trt$$

where

Tr = slope of ramp portion, and

$$u(t - t_a) = \begin{cases} 0 & t < t_a \\ 1 & t \geq t_a \end{cases}$$

If a ramp of negative slope Tr is added at $t = t_2$, the sum produces the desired constant torque region. And if negative slope T_f (corresponding to the drop off portion) is added at $t = t_3$ the expression for transmitted torque becomes:

$$T(t) = u(t)Trt - u(t-t_2)Tr(t-t_2) - u(t-t_3)T_f\,(t-t_3)$$

This expression is then a model of the entire cycle. It can be seen that for a fixed value of maximum rated torque (the height of the plateau region), and certain values for Tr and T_f, one of the two following simplified models may apply. If the slopes are steep and the total engagement-disengagement interval is long, T(t) looks like a step torque:

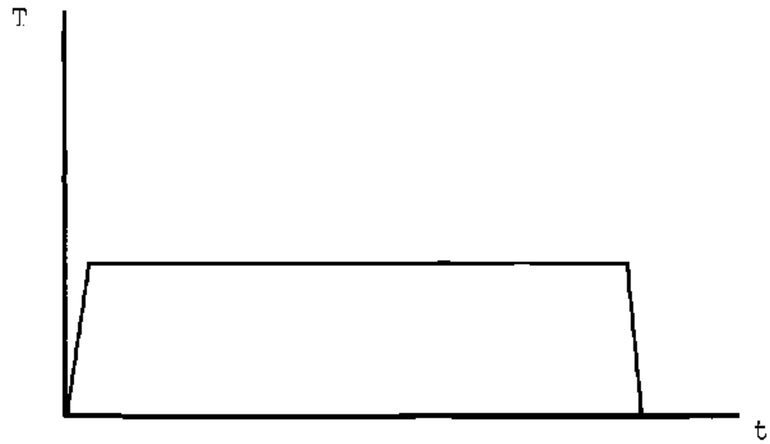


Figure 4. Step Torque Simplified Model.

And for shallow Tr and very short engagements, T(t) approximates a pure ramp torque:

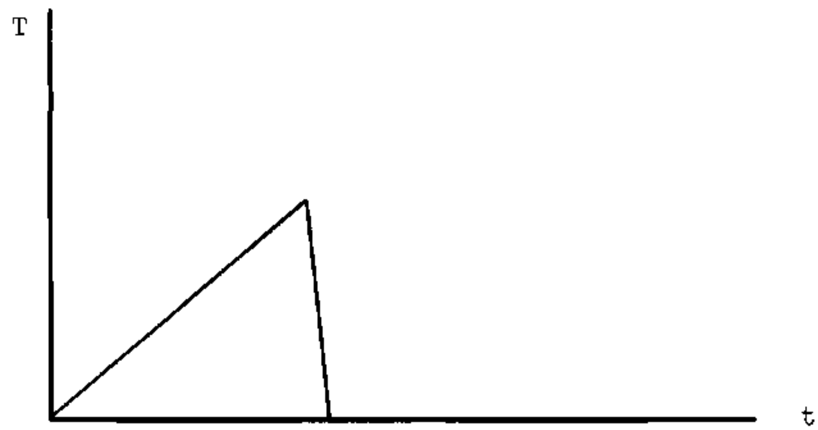


Figure 5. Ramp Torque Simplified Model.

It so happens that the particular system under consideration displays characteristics which tend to place it in the second category. That is, knowing the approximate duration of the engagement interval, and comparing this figure with current clutch response times, indicates that a ramp torque approximation for the clutch transmitted torque is reasonable. Physically, this means that the clutch is being called upon to cycle so rapidly that it never is able to build up to its rated value and operation is confined to the linear buildup region. Throughout the remainder of this thesis emphasis will be placed on a transmitted torque of the form:

$$T(t) = Trt$$

However, since the mechanism might conceivably be applied to a system which tends toward the first category, equations for a step type torque are included at appropriate points.

The decision for a ramp approximation may be argued in another way. In striving to construct a clutch with a step-like response time it becomes possible only to approach this criterion because of natural laws involving system inertia and the elastic behavior of all materials, i.e., a pure step can never be achieved. However, at the other end of the scale, it becomes obvious that the natural tendency of such a device is toward a ramp of some slope governed by these same basic principles. It is then reasonable to assume that the design of a clutch with a prescribed slope is not restrictive.

Load Side Model

General Solution

After the systems are mathematically uncoupled, the load system now appears as below.

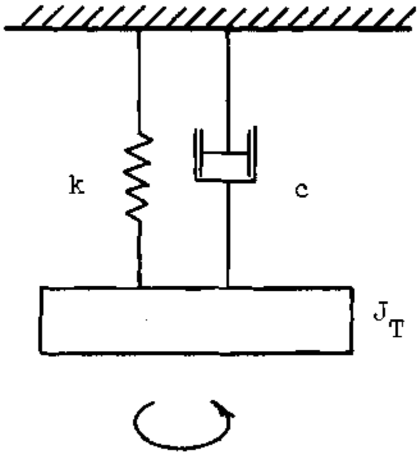


Figure 6. Load Side Schematic

The differential equation of motion for this common damped oscillatory system is given by Newton's second law:

$$J\ddot{\theta} = \sum_i T_i$$

$$J_T\ddot{\theta} = -c\dot{\theta} - k\theta + T(t)$$

or

$$\ddot{\theta} + \frac{c}{J_T}\dot{\theta} + \frac{k}{J_T}\theta = \frac{1}{J_T}T(t)$$

and since

$$\frac{c_c}{J_T} = 2\omega_n \qquad \frac{c}{c_c} = \zeta \qquad \omega_n = \sqrt{\frac{k}{J_T}}$$

the differential equation becomes

$$\ddot{\theta} + 2\zeta\omega_n\dot{\theta} + \omega_n^2\theta = \frac{\omega_n^2}{k}T(t)$$

Now, taking the Laplace transform $(L[\theta(t)] = \theta(s) = \bar{\theta})$, one obtains

$$[s^2\bar{\theta} - s\theta(0) - \dot{\theta}(0)] + 2\zeta\omega_n[s\bar{\theta} - \theta(0)] + \omega_n^2\bar{\theta} = \frac{\omega_n^2}{k}T(s)$$

Rearranging,one obtains for the operational output $\bar{\theta}(s)$:

$$\bar{\theta}(s) = \frac{(s + 2\zeta\omega_n)\theta(0)}{s^2 + 2\zeta\omega_n s + \omega_n^2} + \frac{\dot{\theta}(0)}{s^2 + 2\zeta\omega_n s + \omega_n^2} + \frac{\omega_n^2/k \quad \bar{T}(s)}{s^2 + 2\zeta\omega_n s + \omega_n^2} \quad (1)$$

Utilizing the concept of transfer functions, equation (1) is in a convenient form for analysis. The expression on the right hand side is seen to be the sum of three transfer functions giving the relationship between the operational output $\bar{\theta}(s)$ and initial displacement, initial velocity, and the forcing function. Owing to the linearity of the differential equation these three inputs may be considered separately and their individual responses summed later to obtain the total response.

With this capability it is convenient to consider two regions of operation for the system, that is, the free oscillatory region and the forced oscillatory region. A graph of the proposed motion shows free vibration to take place over time intervals $[0 \; t_1]$ and $[t_2 \; \tau]$ while forced vibration occurs over $[t_1 t_2]$.

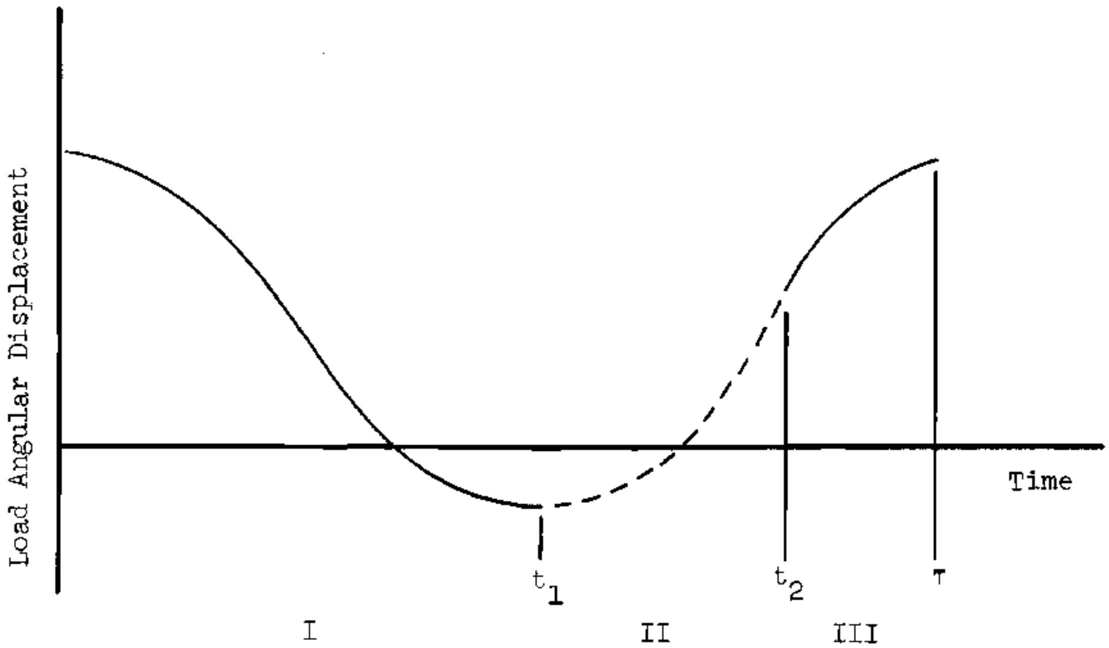


Figure 7. Three Stages of Motion.

It is evident that the first and third stages of motion are one and the same and are governed by the same equations.

First then, the response will be determined for stages I and III, and then the motion in stage II may be found by merely adding the forcing response.

Damped Free Oscillatory Motion

The equation for the operational output after deleting the forcing term becomes:

$$\bar{\theta}(s) = \frac{(s + 2\zeta\omega_n)\theta(0)}{s^2 + 2\zeta\omega_n s + \omega_n^2} + \frac{\dot{\theta}(0)}{s^2 + 2\zeta\omega_n s + \omega_n^2} \tag{2}$$

At this point, it was assumed that the system would always be underdamped, i.e., $\zeta < 1$. The reason being that mechanically it was desirable to have some clear cut point at which to engage the clutch. The point of zero velocity and reversal of direction present in underdamped systems released from zero velocity and a positive displacement satisfies this requirement. Also, the physics of clutch operation dictate adding energy only in one direction while it is engaged, i.e., the transmission interval must take place entirely on one side or the other of this velocity reversal point. Since it is always possible to make a system underdamped by increasing the spring constant k or increasing the inertia, the above assumption is a reasonable one.

Thus, with $\zeta < 1$ equation (2) may be written

$$\bar{\theta}(s) = \theta(0)\left[\frac{(s + \zeta\omega_n)}{(s + \zeta\omega_n)^2 + \omega_d^2} + \frac{\zeta}{\sqrt{1 - \zeta^2}} \frac{\omega_d}{(s + \zeta\omega_n)^2 + \omega_d^2}\right]$$

$$+ \dot{\theta}(0)\left[\frac{1}{\omega_d} \frac{\omega_d}{(s + \zeta\omega_n)^2 + \omega_d^2}\right]$$

where $\omega_d = \omega_n \sqrt{1 - \zeta^2}$, the damped natural frequency. Then taking the inverse transform obtain the equation of motion:

$$\theta(t) = e^{-\zeta\omega_n t} \{\theta(0)[\cos(\omega_d t) + \frac{\zeta}{\sqrt{1 - \zeta^2}} \sin(\omega_d t)] \tag{3}$$

$$+ \dot{\theta}(0) [\frac{1}{\omega_d} \sin(\omega_d t)]\}$$

This equation, then, applies to stages I and III when evaluated at the appropriate initial conditions.

Damped Forced Oscillatory Motion

Repeating equation (1), in Laplace transforms the equation governing forced motion is:

$$\bar{\theta}(s) = \frac{(s + 2\zeta\omega_n)\theta(0)}{s^2 + 2\zeta\omega_n s + \omega_n^2} + \frac{\dot{\theta}(0)}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$$+ \frac{\omega_n^2/k \quad \bar{T}(s)}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

and on taking the inverse transform

$$\theta(t) = e^{-\zeta\omega_n t}\{\theta(0)[\cos(\omega_d t) + \frac{\zeta}{\sqrt{1-\zeta^2}} \sin(\omega_d t)] \tag{4}$$

$$+ \dot{\theta}(0)[\frac{1}{\omega_d} \sin(\omega_d t)]\} + A(t) * T(t)$$

where

$$A(t) = L^{-1}\{\frac{\omega_n^{\,2}/k}{s^2 + 2\zeta\omega_n s + \omega_n^{\,2}}\} \text{ , and}$$

$*$ indicates the convolution integral.

The convolution method is always valid, of course, but for the two cases to be examined next it is simpler to take the inverse transform of the entire applied torque term directly as it appears in the operational output above.

The Response to a Step Torque. Consider the step torque

$$T(t) = T_0 u(t)$$

then

$$\bar{T}(s) = \frac{T_0}{s}$$

and

$$\bar{\theta}_T(s) = \frac{\omega_n^{\ 2}}{k} \; \frac{T_0}{s(s^2 + 2\zeta\omega_n s + \omega_n^{\ 2})}$$

For damping less than critical

$$\bar{\theta}_T(s) = \frac{\omega_n^{\ 2}}{k} T_0 \; \{ \frac{1}{s[(s + \zeta\omega_n)^2 + \omega_d^{\ 2}]} \}$$

The method of partial fractions yields

$$\bar{\theta}_T(s) = \frac{T_0}{k} \{ \frac{1}{s} - \frac{s + 2\zeta\omega_n}{(s + \zeta\omega_n)^2 + \omega_d^{\ 2}} \}$$

Further separation of terms yields

$$\bar{\theta}_T(s) = \frac{T_0}{k} \{ \frac{1}{s} - \frac{s + \zeta\omega_n}{(s + \zeta\omega_n)^2 + \omega_d^{\ 2}} - \frac{\zeta}{\sqrt{1 - \zeta^2}} \; \frac{\omega_d}{(s + \zeta\omega_n)^2 + \omega_d^{\ 2}} \}$$

Then, taking the inverse transform one obtains

$$\theta_T(t) = \frac{T_0}{k} \{ 1 - e^{-\zeta\omega_n t} [\cos\,(\omega_d t) + \frac{\zeta}{\sqrt{1-\zeta^2}} \sin(\omega_d t)]\} \qquad (5)$$

Equation (5) then, represents the value of the convolution integral to be substituted into equation (4) if the system is subjected to a step torque at $t = 0$.

The Response to a Ramp Torque. Consider the ramp torque

$$T(t) = Trt\ u(t)$$

then

$$\bar{T}(s) = \frac{Tr}{s^2}$$

$$\bar{\theta}_T(s) = \frac{\omega_n^2}{k} \frac{Tr}{s^2(s^2 + 2\zeta\omega_n s + \omega_n^2)}$$

For damping less than critical

$$\bar{\theta}_T(s) = \frac{\omega_n^2}{k} Tr \left\{\frac{1}{s^2[(s + \zeta\omega_n)^2 + \omega_d^2]}\right\}$$

In a similar fashion to that used for step input, partial fractions and rearrangement yields:

$$\bar{\theta}_T(s) = \frac{Tr}{k} \left\{\frac{1}{s^2} - \frac{2\zeta}{\omega_n} \left[\frac{1}{s} - \frac{s + \zeta\omega_n}{(s + \zeta\omega_n)^2 + \omega_d^2}\right.\right.$$

$$\left.\left. - \frac{\zeta - \frac{1}{2\zeta}}{\sqrt{1 - \zeta^2}} \frac{\omega_d}{(s + \zeta\omega_n)^2 + \omega_d^2}\right]\right\}$$

Then taking the inverse transform

$$\theta_T(t) = \frac{Tr}{k}\{t - \frac{2\zeta}{\omega_n}[1 - e^{-\zeta\omega_n t}(\cos(\omega_d t) + \tag{6}$$

$$+ \frac{\zeta - \frac{1}{2\zeta}}{\sqrt{1 - \zeta^2}} \sin(\omega_d t))]\}$$

Equation (6) then, represents the value of the convolution integral to be substituted into equation (4) if the system is subjected to a ramp torque at $t = 0$.

These, then, are the general equations for a system of this type. If, now, initial conditions are chosen and the equations governing motion in the separate stages of a cycle are required to match at clutch engagement and disengagement points, the complete equations of motion will result.

Complete Solution

Stage I. Initially it is required that the load be angularly displaced by Θ and at rest, i.e.,

$$\theta(0) = \Theta \qquad \dot{\theta}(0) = 0.$$

Θ is determined such that if released from this configuration, the system will swing through the desired arc (270°) before coming to rest again at the velocity reversal point.

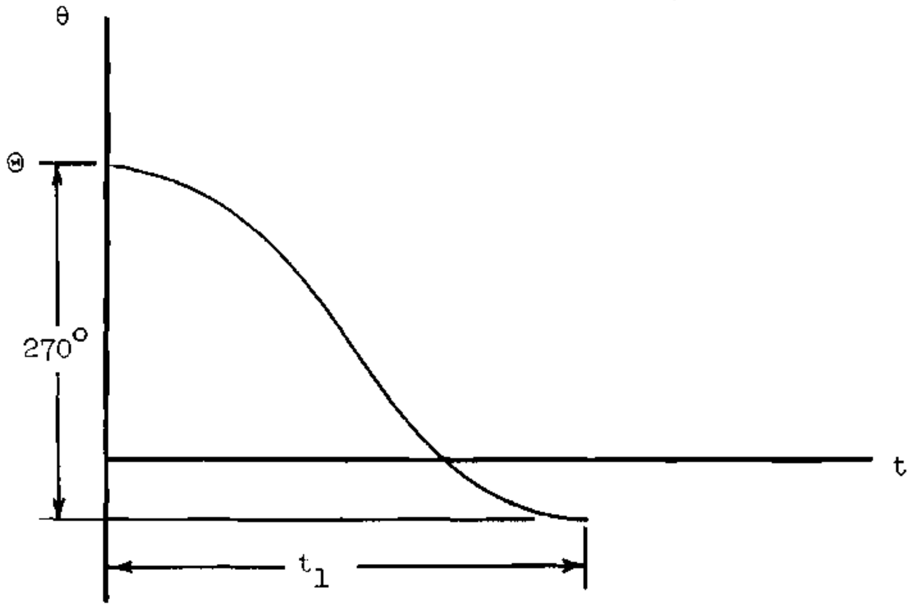


Figure 8. Determination of Θ and t_1.

This is obviously a function of the values of damping, spring constant, and system inertia only and may be solved for directly. Therefore, from equation (3), the motion in stage I is given by

$$\theta_I(t) = \Theta e^{-\zeta\omega_n t}\left[\cos(\omega_d t) + \frac{\zeta}{\sqrt{1-\zeta^2}}\sin(\omega_d t)\right] \qquad (7)$$

<u>Stage II</u>. The equation of motion which takes over in stage II must have initial conditions equal to the final conditions in stage I. If the clutch engagement is taken to be at $t = t_1$, then these conditions will be

$$\theta_I(t=t_1) = \Theta e^{-\zeta\omega_n t_1}[\cos(\omega_d t_1) + \frac{\zeta}{\sqrt{1-\zeta^2}} \sin(\omega_d t_1)]$$

$$\dot{\theta}_I(t=t_1) = -\Theta e^{-\zeta\omega_n t_1}[\frac{\omega_n^2}{\omega_d} \sin(\omega_d t_1)]$$

If in stage II, time is measured by $t*$, then $t* = 0$ when $t = t_1$. Then the "initial" conditions are given by

$$\theta_{II}(t*=0) = \theta_I(t=t_1)$$

$$\dot{\theta}_{II}(t*=0) = \dot{\theta}_I(t=t_1)$$

Substituting these values in equation (4) along with the response to a ramp torque equation (6), obtain the equation of motion in stage II.

$$\theta_{II}(t*) = e^{-\zeta\omega_n t*} \{[\Theta e^{-\zeta\omega_n t_1}(\cos(\omega_d t_1) + \frac{\zeta}{\sqrt{1-\zeta^2}} \sin(\omega_d t_1))]$$

$$\cdot [\cos(\omega_d t^*) + \frac{\zeta}{\sqrt{1-\zeta^2}} \sin(\omega_d t^*)] +$$

$$+ [-\Theta e^{-\zeta\omega_n t_1}(\frac{\omega_n^2}{\omega_d} \sin(\omega_d t_1))] \times [\frac{1}{\omega_d} \sin(\omega_d t^*)]\}$$

$$+ \frac{Tr}{k} \{t^* - \frac{2\zeta}{\omega_n} [1 - e^{-\zeta\omega_n t^*} (\cos(\omega_d t^*) + \frac{\zeta - \frac{1}{2\zeta}}{\sqrt{1-\zeta^2}} \sin(\omega_d t^*))]\}$$

After rearranging, simplifying and substituting $t^* = t-t_1$, obtain:

$$\theta_{II}(t) = \Theta e^{-\zeta\omega_n t} [\cos(\omega_d t) + \frac{\zeta}{\sqrt{1-\zeta^2}} \sin(\omega_d t)] \qquad (8)$$

$$+ \frac{Tr}{k} \{(t-t_1) - \frac{2\zeta}{\omega_n} [1 - e^{-\zeta\omega_n (t-t_1)} (\cos[\omega_d (t-t_1)]$$

$$+ \frac{\zeta - \frac{1}{2\zeta}}{\sqrt{1-\zeta^2}} \sin [\omega_d (t-t_1)])]\}$$

Stage III. Proceeding in a similar manner, obtain the initial conditions for stage III motion as

$$\theta_{II}(t=t_2) = \Theta e^{-\zeta\omega_n t_2}[\cos(\omega_d t_2) + \frac{\zeta}{\sqrt{1-\zeta^2}} \sin(\omega_d t_2)]$$

$$+ \frac{Tr}{k}\{(t_2-t_1) - \frac{2\zeta}{\omega_n}[1-e^{-\zeta\omega_n(t_2-t_1)}(\cos[\omega_d(t_2-t_1)]$$

$$+ \frac{\zeta - \frac{1}{2\zeta}}{\sqrt{1-\zeta^2}} \sin [\omega_d(t_2-t_1)]])]\}$$

$$\dot{\theta}_{II}(t=t_2) = \frac{Tr}{k}\{1-\zeta e^{-\zeta\omega_n(t_2-t_1)}[\frac{1}{\sqrt{1-\zeta^2}}\sin[\omega_d(t_2-t_1)]$$

$$+ \frac{1}{\zeta} \cos [\omega_d(t_2-t_1)]]]\} - \Theta \frac{\omega_n}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t_2} \sin(\omega_d t_2)$$

If in stage III, time is measured by t^{**}, then $t^{**} = 0$ when $t = t_2$. The initial conditions are given by

$$\theta_{III}(t^{**} = 0) = \theta_{II}(t=t_2)$$

$$\dot{\theta}_{III}(t^{**} = 0) = \dot{\theta}_{II}(t = t_2)$$

These are the appropriate initial conditions to be substituted in the general equation of motion, equation (3). If this is done along with the knowledge that $t^{**} = t - t_2$ obtain

$$\theta_{III}(t) = e^{-\zeta\omega_n(t-t_2)}\{[\Theta e^{-\zeta\omega_n t_2}(\cos(\omega_d t_2) + \frac{\zeta}{\sqrt{1-\zeta^2}}\sin(\omega_d t_2))$$

$$+ \frac{Tr}{k}\{(t_2-t_1) - \frac{2\zeta}{\omega_n}[1-e^{-\zeta\omega_n(t_2-t_1)}(\cos[\omega_d(t_2-t_1)] +$$

$$+ \frac{\zeta - \frac{1}{2\zeta}}{\sqrt{1-\zeta^2}} \sin[\omega_d(t_2-t_1)])]\}]$$

$$\cdot [\cos[\omega_d(t-t_2)] + \frac{\zeta}{\sqrt{1-\zeta^2}} \sin[\omega_d(t-t_2)]]$$

$$+ [\frac{Tr}{k}\{1-\zeta e^{-\zeta\omega_n(t_2-t_1)}[\frac{1}{\sqrt{1-\zeta^2}} \sin[\omega_d(t_2-t_1)] +$$

$$+ \frac{1}{\zeta}\cos[\omega_d(t_2-t_1)]]\}$$

$$- \Theta\frac{\omega_n}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t_2} \sin(\omega_d t_2)]\cdot[\frac{1}{\omega_d}\sin[\omega_d(t-t_2)]]\}$$

Equations (7) and (8), along with the above equation, give the motion in stages I, II, and III, respectively. These three equations assure that final and initial conditions at clutch engagement and disengagement points are properly matched.

The problem of finding t_2 and Tr such that the desired motion results is best solved using numerical analysis techniques and considering only equations (7) and (8), however. Since stages I and III are in actuality the same, considerable reduction in complexity is gained by eliminating a separate stage III equation. Further discussion of this problem and the numerical solution procedure followed are contained in Chapter III. Suffice it to say at this point that the solution results in a value for the ramp torque slope and the time interval over which it is to be applied, so that if repeated cyclically, a smooth continuous motion results.

Induction Motor Characteristics

Initially it is desirable to specify a standard type of electric motor. This is a cost consideration primarily but also reflects availability, simplicity and reliability. A survey of current motor literature reveals the almost infinite variety and specialization of motors. Variations in rotor and stator winding, slip characteristics, starting circuit and mechanical construction combine to offer precise characteristics for a particular job. However, these all go hand-in-hand with increased cost and complexity leaving certain common varieties as the most practical. Motor classifications and standards have been established by the National Electrical Manufacturers Association (NEMA). These standards, NEMA Standards Publication M61-1967, Motors and

Generators, cover both electrical performance and mechanical configuration and serve as a guide to both user and manufacturer. In picking a "standard" motor the designer is restricted to certain performance characteristics rather than being able to specify them.

Certain obvious categories such as voltage, frequency and power rating narrow the choice somewhat but final selection is determined by the motor performance characteristics which are generally presented as a plot of speed versus torque. Motor torque is a strong function of shaft speed and it is this functional relationship that distinguishes among the various NEMA classifications. Basic curves indicate the torque developed by a motor at any given speed from its starting point, past the full load point, to a theoretical zero torque point. Shown below are typical curves for three of the most popular NEMA design categories:

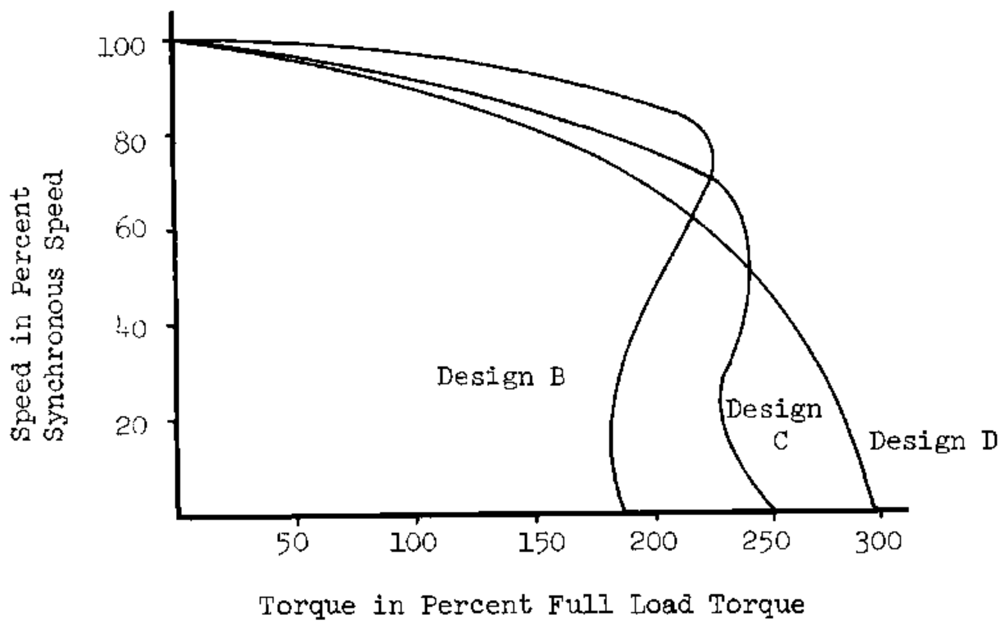


Figure 9. Torque Versus Speed for Three NEMA Motor Designs.

Important points on a speed-torque curve are:

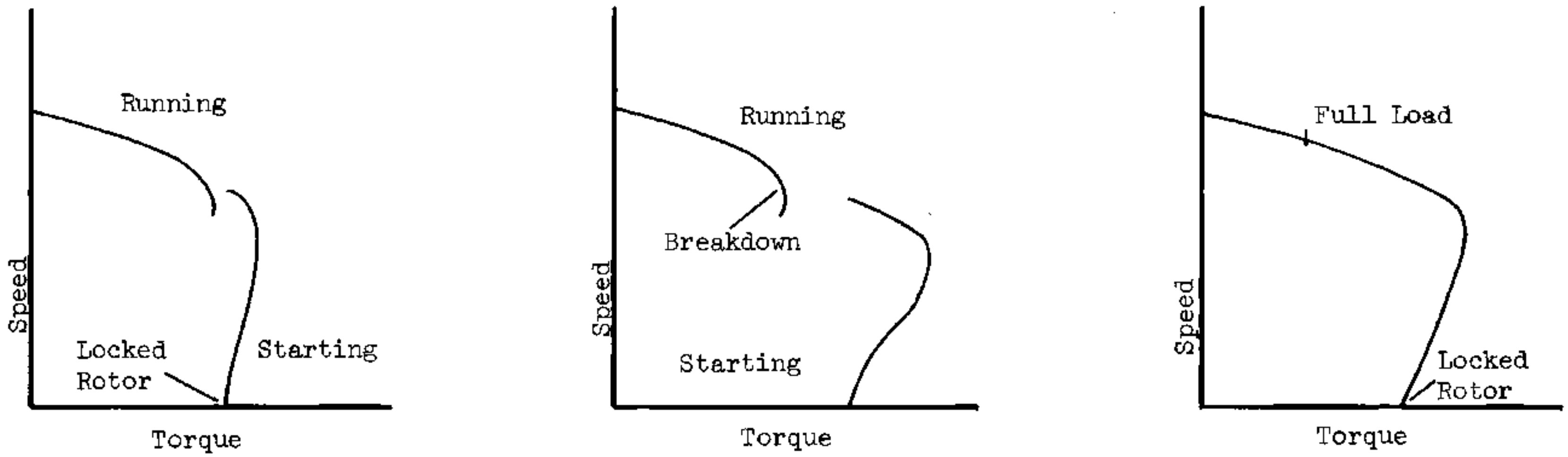
(1) Full-load torque - the point at which rated power is developed when rated voltage is applied to the motor.

(2) Breakdown torque - the point at the "knee" of the curve where any additional load torque causes the motor to decelerate rapidly.

(3) Locked-rotor torque - the initial torque available when the motor is energized at standstill.

Fractional horsepower motors, generally, have only one type of performance - NEMA Design B. The mechanism considered in this thesis could theoretically operate at any power range and capacity, however, the main interest at this time is in powering machinery requiring less than one horsepower. Therefore, throughout the remaining work the motor is taken to be Design B, with the understanding that larger power requirements may also be furnished by Design B integral horsepower motors. Also, for that matter, the appropriate curve for any type motor may be substituted at a later date to adapt the mechanism to a different load system.

Once a particular class of motors is chosen, subdivisions according to starting circuit are available, Figure 10. After breakaway, the motor accelerates along the starting curve to a speed at which a centrifugal switch cuts out the starting winding, capacitor, etc. and acceleration continues along the "new" running curve until the speed is reached where load requirement and motor output are matched. The various varieties of starting circuits produce higher and higher starting torques at increased cost and complexity. Again, the choice is made for the split-phase because it



(a) Split Phase (b) Capacitor Start (c) Three Phase

NEMA Design B Fractional Horsepower

Figure 10. Torque Versus Speed for Three Motor Starting Circuits.

is the simplest and most common. With this type of operation, acceleration on the starting curve stops at approximately 75 percent of synchronous speed, where the latter is defined to be that speed at which the motor would operate if the rotor turned at the exact speed of the rotating magnetic field. Since rotor speed must always lag somewhat behind this field to experience any applied torque at all, the synchronous speed represents an upper bound on motor speed. Running operation then is always less than synchronous and this difference at any point on the curve is defined as slip.

The operating point determined by matched load and motor requirements in turn determines the power the motor must supply, i.e., a certain torque at a certain speed fixes the horsepower. It is evident that a given motor may operate at a horsepower level which is different from its rating, i.e., a 0.5 Hp motor may be required to run at 0.6 - 0.7 Hp. Most motors are designed so that an overload of some fixed amount is allowable without damaging the motor. This is specified in terms of a service factor which when multiplied by the rated power gives an upper limit on loading. In all cases, however, operation should be confined to the running portion of the curve - even with a fluctuating load as is the case under consideration.

Now, taking a typical motor of the type selected and inverting the curve to obtain torque versus speed, one obtains the following plot, Figure 11. It is necessary to obtain a functional relationship for motor torque in terms of shaft speed. Fortunately, the portion of the running curve to the right of breakdown speed is very nearly linear, and since this is the region where operation will be confined,

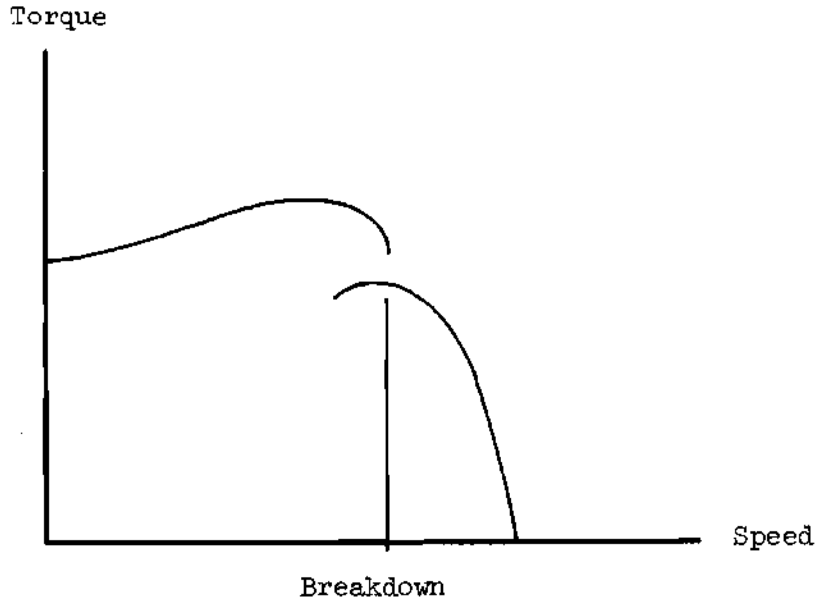


Figure 11. Torque Versus Speed for Selected Motor.

an equation of the form

$$y = mx + b$$

for a straight line can be obtained. Designating the slope as α and the intercept at zero speed as TQ, the equation is

$$T(\dot{\varphi}) = \alpha\dot{\varphi} + TQ$$

As can be seen in Figure 12, the curve is an excellent approximation to the real curve in the operating range but care must be taken so that breakdown speed is never reached because considerable deviation from the real curve exists below this point. Also, it is obvious that start-up and acceleration performance must be obtained elsewhere.

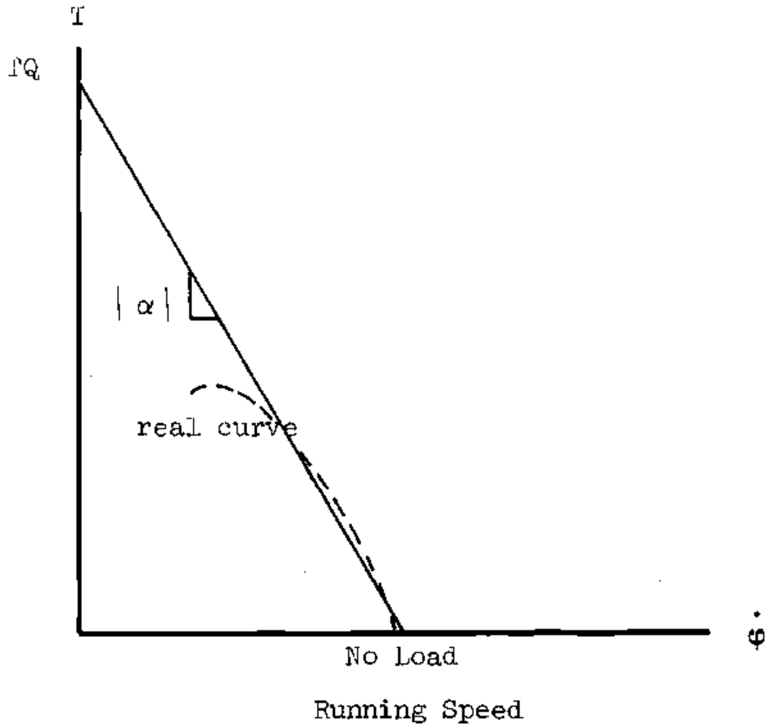
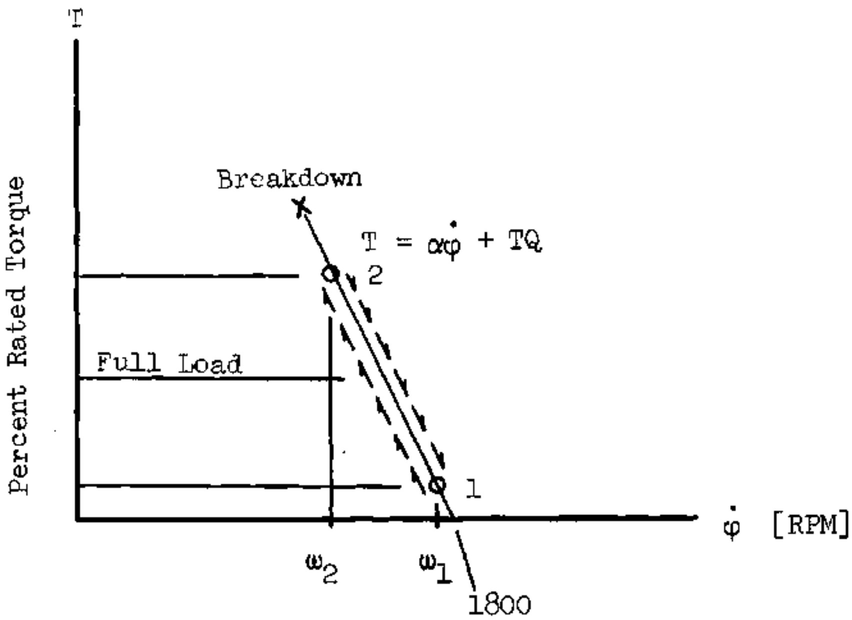


Figure 12. Linear Approximation of Torque Versus Speed Characteristics.

For a cycle wherein the motor is clutched into a load for a portion of the cycle and then declutched and allowed to regain running speed for the remainder of the work cycle, a loop is traversed on this graph. With point 1 corresponding to the initiation of the cycle with engagement of the clutch and point 2 corresponding to the disengagement, the work cycle would appear as in Figure 13.



Note: 1800 RPM is taken to be the no load running speed

Figure 13. Cycle of Motor Operation

Drive Side Model

After uncoupling, the drive system appears as below:

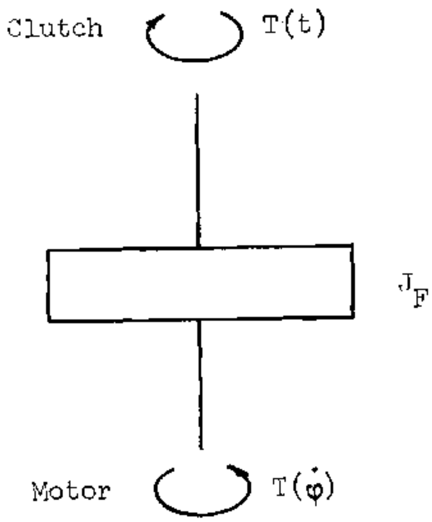


Figure 14. Schematic of Drive Side.

where the entire drive-side inertia is lumped into J_F. The torque transmitted through the clutch, $T(t) = Trt$, and the torque supplied by the motor, $T(\dot{\varphi}) = \alpha\dot{\varphi} + TQ$, have been derived previously and constitute the force system acting at any time.

It is evident from Figure 7 (repeated here as the first section of Figure 15) that the duty cycle of the motor will reflect the varying load. This is a plot of the torque-demand from the load only, however, as specified by the solution of the equations of motion for the load side. The motor will not be running load free in the declutched intervals, for it is at this time that it must supply the energy given up by the flywheel. Again, the motion may be divided into two regions:

(1) During which the motor is accelerating back to running speed, and

(2) During which time the clutch is engaged.

Acceleration Interval

When the clutch is disengaged, the motor must exert torque T to accelerate its inertia (armature plus flywheel) from some ω_2 back to ω_1, the running speed. For a pure inertial load, writing Newton's second law:

$$\sum_{i=1}^{n} T_i = J\ddot{\varphi}$$

For equivalent inertia J_F and motor torque T,

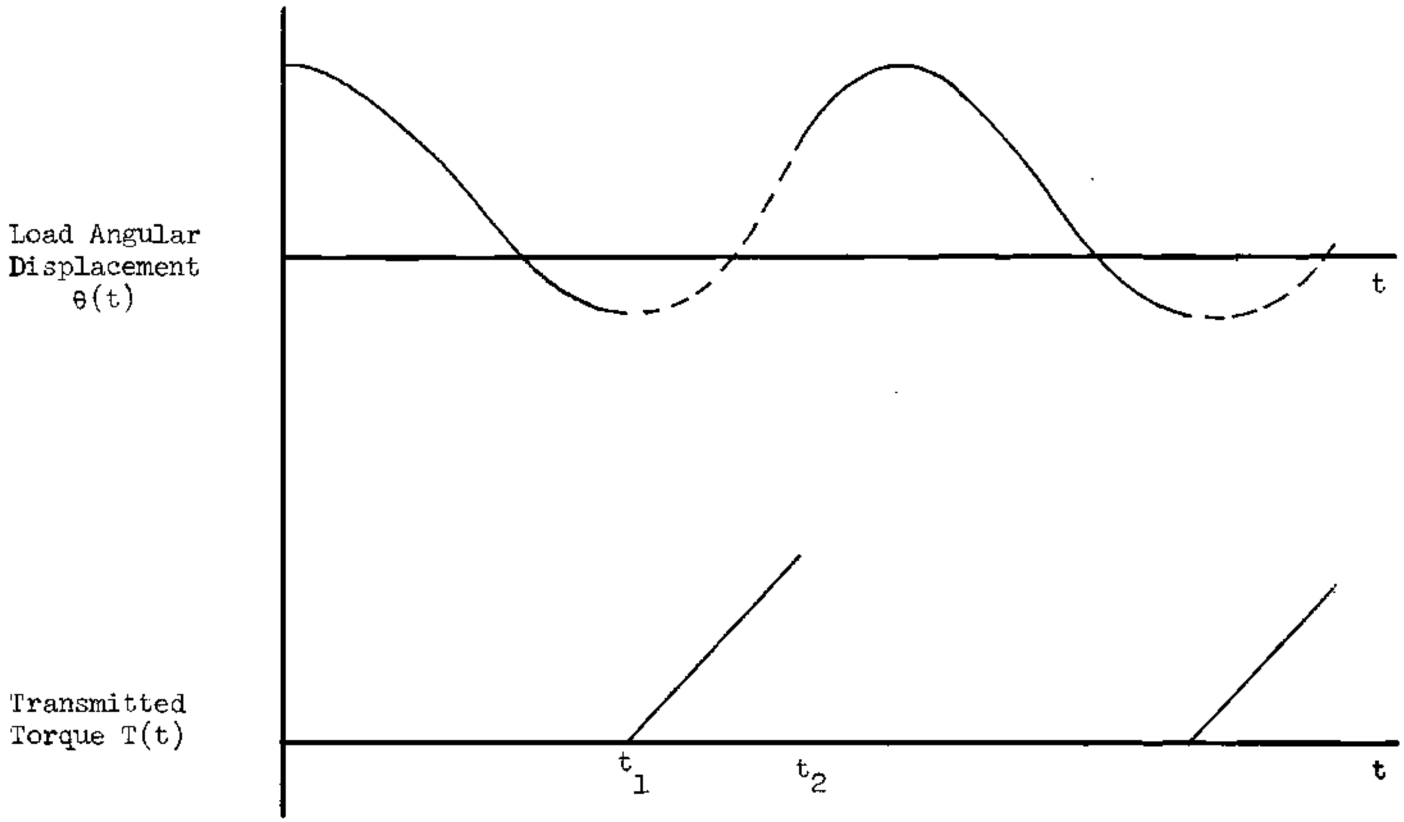


Figure 15. Motor Duty Cycle.

$$J_F\ddot{\varphi} = T$$

but $T = T(\dot{\varphi}) = \alpha\dot{\varphi} + TQ$, therefore

$$J_F\ddot{\varphi} = \alpha\dot{\varphi} + TQ$$

giving the differential equation

$$\ddot{\varphi} - \frac{\alpha}{J_F}\,\dot{\varphi} = \frac{TQ}{J_F}$$

Now, taking the Laplace transform ($L[\varphi(t)] = \varphi(s) = \bar{\varphi}$), one obtains

$$s^2\bar{\varphi}(s) - s\varphi(0) - \dot{\varphi}(0) - \frac{\alpha}{J_F}\,[s\bar{\varphi}(s) - \varphi(0)] = \frac{TQ}{J_F}\;\frac{1}{s}$$

Since $\varphi(0)$ is arbitrary, it will be taken to be zero, and the angular velocity at disengagement will be called ω_2. Rearrangement into operational form yields:

$$\bar{\varphi}(s) = \frac{TQ}{J_F}\;\frac{1}{s^2(s - \frac{\alpha}{J_F})} + \omega_2\,\frac{1}{s(s - \frac{\alpha}{J_F})}$$

with the help of partial fractions this may be rewritten as

$$\bar{\varphi}(s) = \frac{TQ}{J_F}\{-(\frac{J_F}{\alpha})^2 \frac{1}{s} - (\frac{J_F}{\alpha}) \frac{1}{s^2} + (\frac{J_F}{\alpha})^2 \frac{1}{s - \frac{\alpha}{J_F}}\} +$$

$$+ \omega_2 \{- \frac{J_F}{\alpha} \frac{1}{s} + (\frac{J_F}{\alpha}) \frac{1}{s - \frac{\alpha}{J_F}}\}$$

Then, taking the inverse transform and rearranging one obtains

$$\varphi(t) = e^{\frac{\alpha}{J_F} t} \{\frac{TQ}{J_F} (\frac{J_F}{\alpha})^2 + \omega_2(\frac{J_F}{\alpha})\} - \frac{TQ}{\alpha} t - [\frac{TQ}{J_F} (\frac{J_F}{\alpha})^2 + \omega_2(\frac{J_F}{\alpha})]$$

The quantity that is needed, however, is velocity, so differentiation with respect to time yields

$$\dot{\varphi}(t) = e^{\frac{\alpha}{J_F} t} \{\frac{TQ}{\alpha} + \omega_2\} - \frac{TQ}{\alpha} \tag{9}$$

This equation gives the speed fluctuation during the acceleration interval.

Transmission Interval

When the clutch is engaged, the transmitted torque build-up is prescribed to be a ramp given by Trt. Then from Newton's second law:

$$J_F\ddot{\varphi} = \alpha\dot{\varphi} + TQ - Trt$$

or

$$\ddot{\varphi} - \frac{\alpha}{J_F} \dot{\varphi} = \frac{TQ}{J_F} - \frac{Tr}{J_F} t$$

Now, taking the Laplace transform

$$s^2\bar{\varphi}(s) - s\varphi(0) - \dot{\varphi}(0) - \frac{\alpha}{J} \{s\bar{\varphi}(s) - \varphi(0)\} =$$

$$= \frac{TQ}{J} \frac{1}{s} - \frac{Tr}{J} \frac{1}{s^2}$$

Again, $\varphi(0)$ is arbitrary and the angular velocity at engagement is called ω_1. Rearrangement and partial fractions as before yield:

$$\varphi(s) = \frac{TQ}{J_F} \{-(\frac{J_F}{\alpha})^2 \frac{1}{s} - (\frac{J_F}{\alpha}) \frac{1}{s^2} + (\frac{J_F}{\alpha})^2 \frac{1}{s - \frac{\alpha}{J_F}} \} +$$

$$+ \omega_1 \{- \frac{J_F}{\alpha} \frac{1}{s} + (\frac{J_F}{\alpha}) \frac{1}{s - \frac{\alpha}{J_F}} +$$

$$- \frac{Tr}{J_F} \{-(\frac{J_F}{\alpha})^3 \frac{1}{s} - (\frac{J_F}{\alpha})^2 \frac{1}{s^2} - (\frac{J_F}{\alpha}) \frac{1}{s^3} \quad (\frac{J_F}{\alpha})^3 \frac{1}{s - \frac{\alpha}{J_F}} \}$$

Taking the inverse transform and rearranging obtain:

$$\varphi(t) = e^{\frac{\alpha}{J_F} t} \{\omega_1(\frac{J_F}{\alpha}) + \frac{TQ}{J_F}(\frac{J_F}{\alpha})^2 - \frac{Tr}{J_F}(\frac{J_F}{\alpha})^3\} + \frac{1}{2}\frac{Tr}{\alpha} t^2 +$$

$$+ \{ \frac{TrJ_F}{\alpha^2} - \frac{TQ}{\alpha} \}t - [\frac{TQJ_F}{\alpha^2} + \omega_1(\frac{J_F}{\alpha}) - \frac{Tr}{J_F}(\frac{J_F}{\alpha})^3]$$

Again, the quantity that is needed is velocity, so differentiation with respect to time yields

$$\dot{\varphi}(t) = e^{\frac{\alpha}{J_F} t} \{\omega_1 + \frac{TQ}{\alpha} - \frac{TrJ_F}{\alpha^2}\} + \frac{Tr}{\alpha} t + [\frac{TrJ_F}{\alpha^2} - \frac{TQ}{\alpha}] \qquad (10)$$

Equation (10) gives the speed fluctuation during the clutched transmission interval.

With the knowledge that α is negative (from the motor torque-speed curve) it is possible to visualize the motor shaft motion, as follows.

Equation (10) shows an essentially exponential form for the speed drop during clutch engagement; Figure 16.

Equation (9) shows a corresponding exponential speed increase during disengagement. Also, the effect of varying J_F becomes clear. These curves satisfy intuition in that increasing flywheel size decreases the speed fluctuation ($\omega_1 - \omega_2$) by "flattening" both curves so that both deceleration and acceleration are slower; Figure 17. After constructing these curves, it becomes evident that ω_1 is not necessarily equal to the no-load running speed ($-TQ/\alpha$). Most likely,

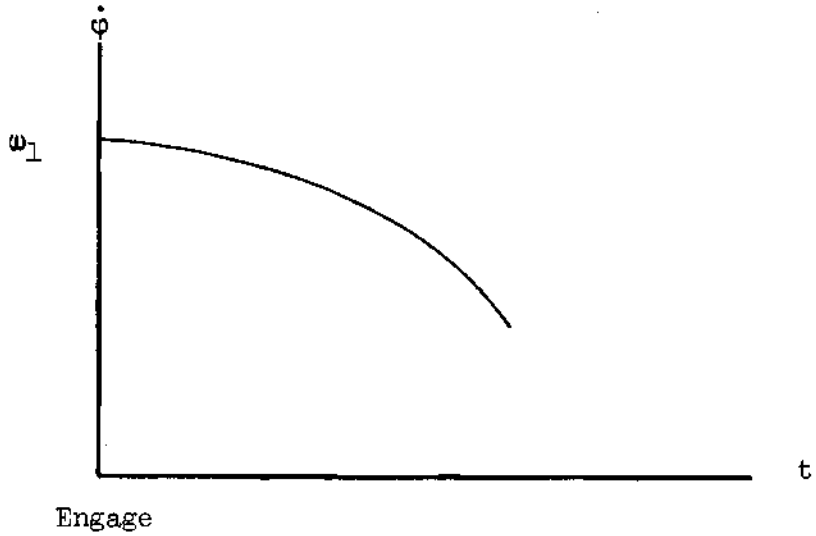


Figure 16. Speed Decrease During Transmission.

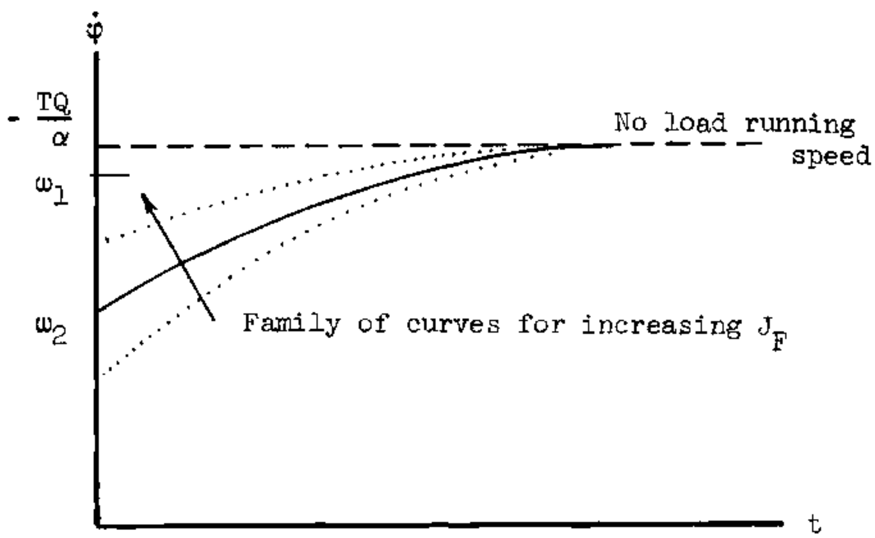


Figure 17. Family of Acceleration Curves.

the operating cycle will take place in the region bounded by the no-load speed and breakdown, i.e., 1800 > ω_1 > ω_2 > Breakdown rpm

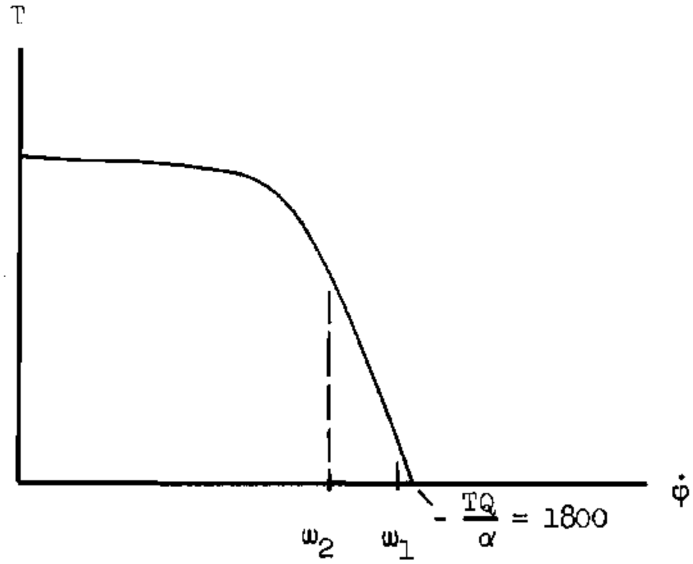


Figure 18. Relative Locations of ω_1 and ω_2.

Again, numerical analysis may be utilized. It is possible to find the equilibrium value of ω_1 such that in the intervals already prescribed by the load side solution (transmission and acceleration intervals fixed) the motor will decrease to some value and then precisely regain ω_1 in time for a new engagement cycle. The low value is then called ω_2. Further details of this approach are given in Chapter III.

If the transmitted torque has the form of a step torque the following development applies during the transmission interval.

$$J_F\ddot{\varphi} = \alpha\dot{\varphi} + TQ - T_0$$

or

$$\ddot{\varphi} - \frac{\alpha}{J_F}\dot{\varphi} = \frac{TQ - T_O}{J_F}$$

This equation has exactly the same form as the differential equation of motion for the acceleration interval, page 37. Therefore, the response to a step torque may be obtained by merely substituting $(TQ - T_O)$ for TQ in the solution, equation (9), giving:

$$\dot{\varphi}(t) = e^{\frac{\alpha}{J_F} t} \{\frac{TQ-T_O}{\alpha} + \omega_1\} - \frac{(TQ - T_O)}{\alpha} \qquad (11)$$

This equation, then, gives the speed fluctuation during the transmission interval if a step torque is used, and replaces equation (10).

<u>Gearing</u>

As will be discussed later, initial solutions considering both sides of the clutch indicated the need for a gear reduction on the drive side. At this point, only the necessary development as pertains to the drive side equations will be considered, forestalling the reasoning behind the addition until Chapter IV.

If a system such as the one illustrated in Figure 19 is utilized, certain basic rules governing torque, speed and inertias across the reduction apply.

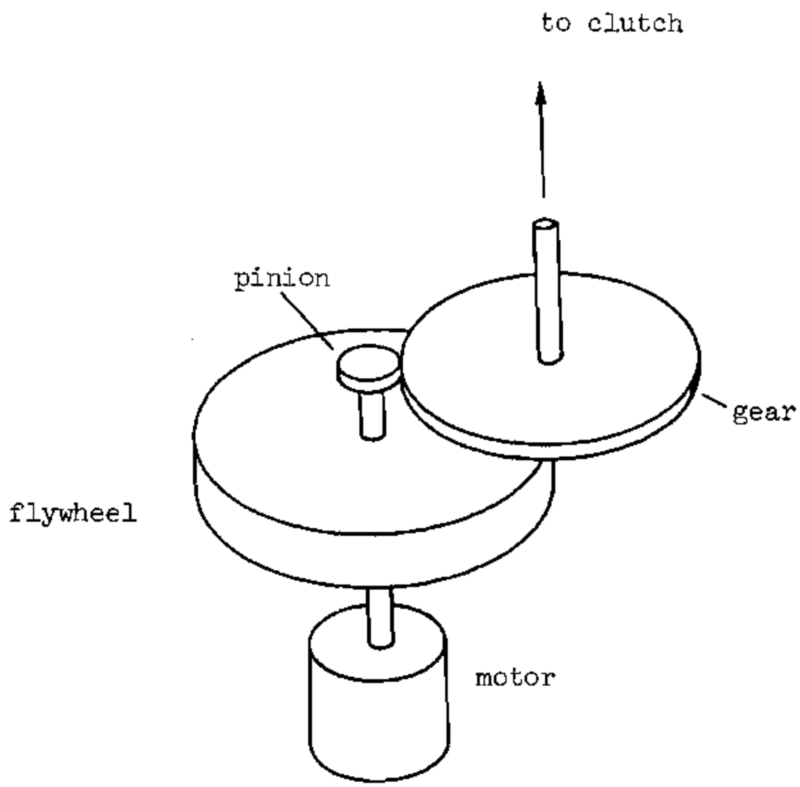


Figure 19. Proposed Gear Reduction Unit.

These rules, stated here without proof, are available from most machine dynamics texts:

(1) Torques on the gear (output) shaft reflect onto the pinion (motor) shaft as their native value divided by the step down ratio.

(2) Inertias on the gear shaft reflect onto the pinion shaft as their native value divided by the step down ratio squared.

(3) The gear shaft speed is the motor speed divided by the step down ratio.

where the stepdown ratio R is given by

$$R = \frac{\text{Gear Radius}}{\text{Pinion Radius}} .$$

Since the motor shaft speed is of prime interest (motor speed fluctuation determines loading and possible stall), it is desirable to reflect these various quantities onto the motor shaft. It is convenient, also, to lump the various inertias into one quantity called J_F on the motor shaft. This permits freedom in selecting a simple value for J_F while permitting the final optimization to resolve J_F into its various components. Accordingly, for the drive side only:

$$J_F = J_{motor} + J_{flywheel} + J_{pinion} + \frac{1}{R^2}(J_{gear} + J_{clutch})$$

Therefore, it is now possible to write the modified version of equation (10) utilizing the above rules:

$$\dot{\varphi}(t) = e^{\frac{\alpha}{J_F} t} \{\omega_1 + \frac{TQ}{\alpha} - \frac{TrJ_F}{R\alpha^2}\} + \frac{Tr}{R\alpha} t + \qquad (12)$$

$$+ [\frac{TrJ_F}{R\alpha^2} - \frac{TQ}{\alpha}]$$

Equation (9), remains unchanged, of course, since J_F contains the various components and Tr is absent. Equations (9) and (12) therefore, comprise the governing equations for the drive side motion.

Power Loading

For a motor which carries a fluctuating duty cycle, as in the mechanism under consideration, it is necessary to compute some indication of the power load at which it is operating. This takes the form of an averaged horsepower, i.e., the mathematical average of the instantaneous computed horsepower throughout one cycle.

Since the motor speed and torque are known throughout a cycle of operation, it is possible to compute the total work done in one cycle as

$$W = \int_{\theta_t}^{\theta_{t+\tau}} T d\theta$$

$$= \int_{t}^{t+\tau} T(\dot{\theta}) \frac{d\theta}{dt} dt$$

$$W = \int_{t}^{t+\tau} T(\dot{\theta}) \dot{\theta} dt$$

where the limits indicate integration over one cycle. If this value is then divided by the time interval over which the motor does work, i.e., the period, a value for the average horsepower results

$$\text{Average Howsepower} = \frac{W}{\tau}$$

It is this value which should agree with the specified full load horsepower rating times any service factor specified by the manufacturer if the motor is to perform satisfactorily.

CHAPTER III

ITERATIVE SOLUTIONS

The problem of satisfying conditions on both sides of the clutch simultaneously was solved iteratively on the digital computer utilizing numerical analysis techniques. The nature of the problem is such that it requires iterations within iterations and a series of nestled loops is necessary. Each loop is assigned an alphabetical name for reference with the computer program. The fixed design parameters, either chosen or dictated before initiating the program steps are the load inertia and damping, the desired arc and period of motion, the motor characteristics, and the flywheel inertia. Again, the load side and the drive side may be examined separately.

Load Side-(Loops B and C)

In designing a mechanism to produce the desired motion it is necessary to find a value of Tr and how long it must be applied. The conditions are that the system coast precisely back to the same configuration as prescribed by the initial conditions, i.e., $\theta = \Theta$ and $\dot{\theta} = 0$, and that the motion have the prescribed period τ.

A Continuous, Periodic Motion - (Loop C)

To assure continuous, periodic motion the clutch disengagement time t_2 must be found for a given Tr. This problem was solved utilizing the Newton-Raphson method for finding the roots of a system of non-linear algebraic equations. Before describing this technique, the following discussion of a simple graphical method which is a helpful preface to the numerical analysis is presented.

Graphical Method. If an initial value for Tr is picked, then from equations (7) and (8), θ versus t may be plotted for regions I and II. The plot might appear as below:

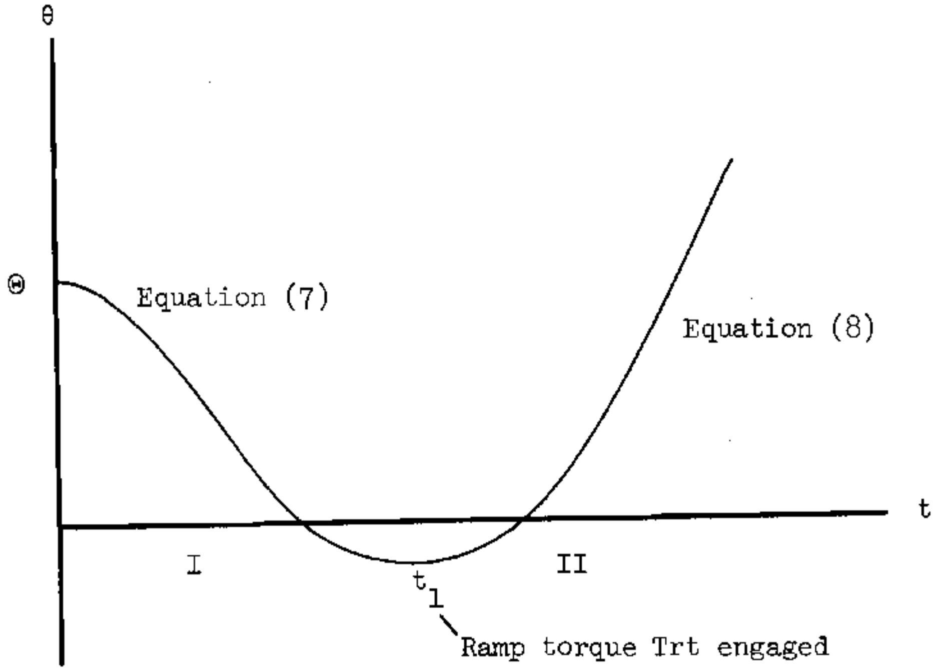


Figure 20. Plot of Stages I and II.

The curve for region II would increase without bound because the applied torque increases linearly with time. It is also known that at some t_2 the external torque will be removed and free vibration will again take over, in fact, the same equation governing region I (equation (7)) will apply. Therefore, the above curve must join the curve in Figure 21, which is obtained by plotting equation (7) and considering negative values of t.

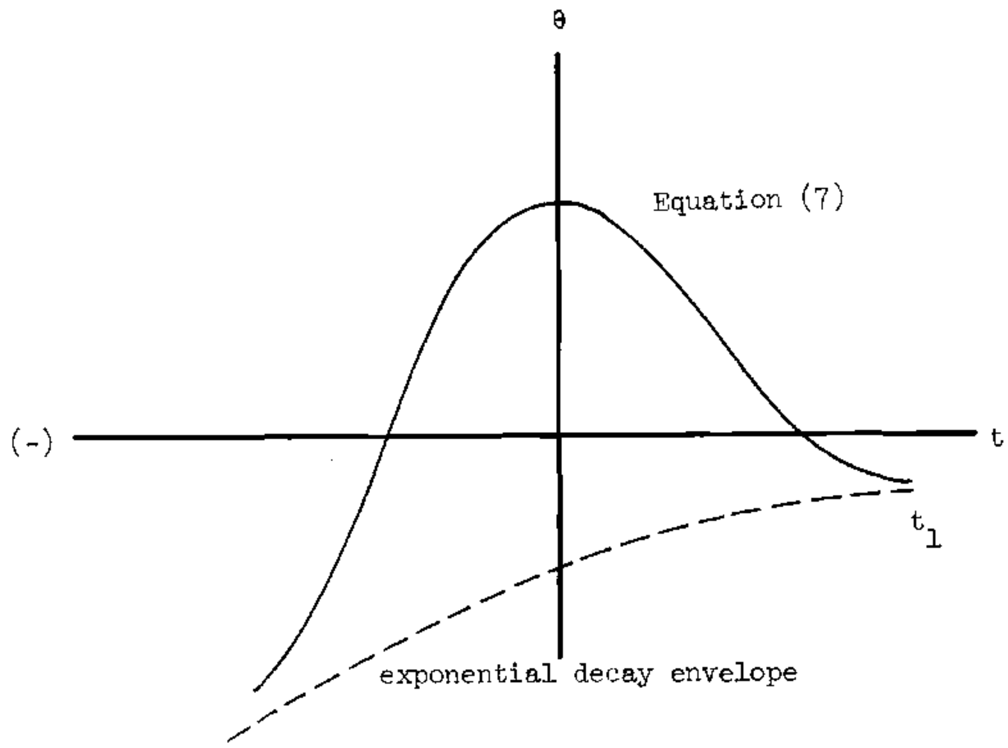


Figure 21. Plot of Stage III.

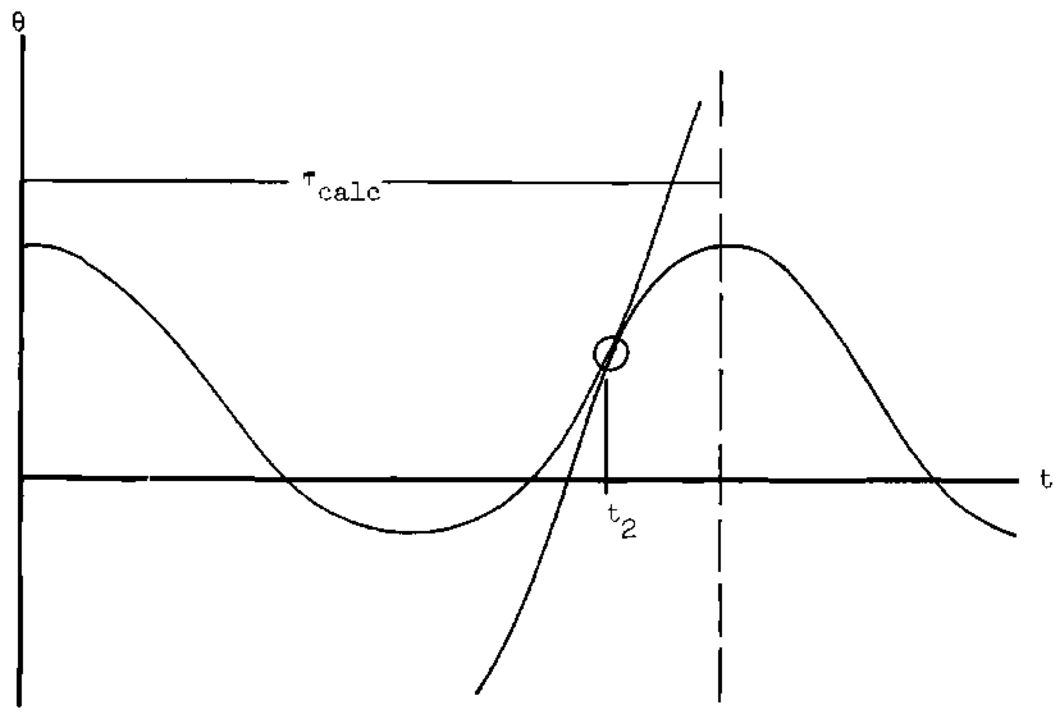


Figure 22. Locating the Point of Tangency.

It is evident then that Figures 20 and 21 could be placed together so that a point of tangency could be found. The point of tangency yields a value for t_2, given a certain Tr, such that the requirement of precisely reaching the same configuration as defined by the initial conditions is assured, and at the same time provides for matched final and initial conditions at the point of disengagement, i.e., tangency. This is illustrated in Figure 22.

Newton-Raphson Method. The Newton-Raphson method may be set up to follow the graphical scheme above utilizing numerical analysis.

For a system of equations

$$F(x,y) = 0$$

$$G(x,y) = 0$$

the N-R method finds successive approximations to the roots x and y by the following formulas:

$$x_n = x_{n-1} + h_{n-1}$$

$$y_n = y_{n-1} + k_{n-1}$$

where h and k satisfy

$$\frac{\partial F(x_{n-1},y_{n-1})}{\partial x} h_{n-1} + \frac{\partial F(x_{n-1},y_{n-1})}{\partial y} k_{n-1} =$$

$$= - F(x_{n-1},y_{n-1})$$

$$\frac{\partial G(x_{n-1},y_{n-1})}{\partial x} h_{n-1} + \frac{\partial G(x_{n-1},y_{n-1})}{\partial y} k_{n-1} =$$

$$= - G(x_{n-1},y_{n-1})$$

This sequence can be set up to find the point of tangency discussed previously. Let the system of equations F and G by given by

$$F(t_2,t_3) = \theta_2(t_2) - \theta_3(t_3)$$

$$G(t_2,t_3) = \dot{\theta}_2(t_2) - \dot{\theta}_3(t_3)$$

where the corresponding illustration is given in Figure 23. The N-R method, then, yields a t_2 and t_3 (t_3 will be negative) such that the displacements are equal and the velocities (slopes) are equal, i.e., the tangency point.

The Newton-Raphson method is very popular due to its rapid rate of convergence and is well suited to this problem. However, it is

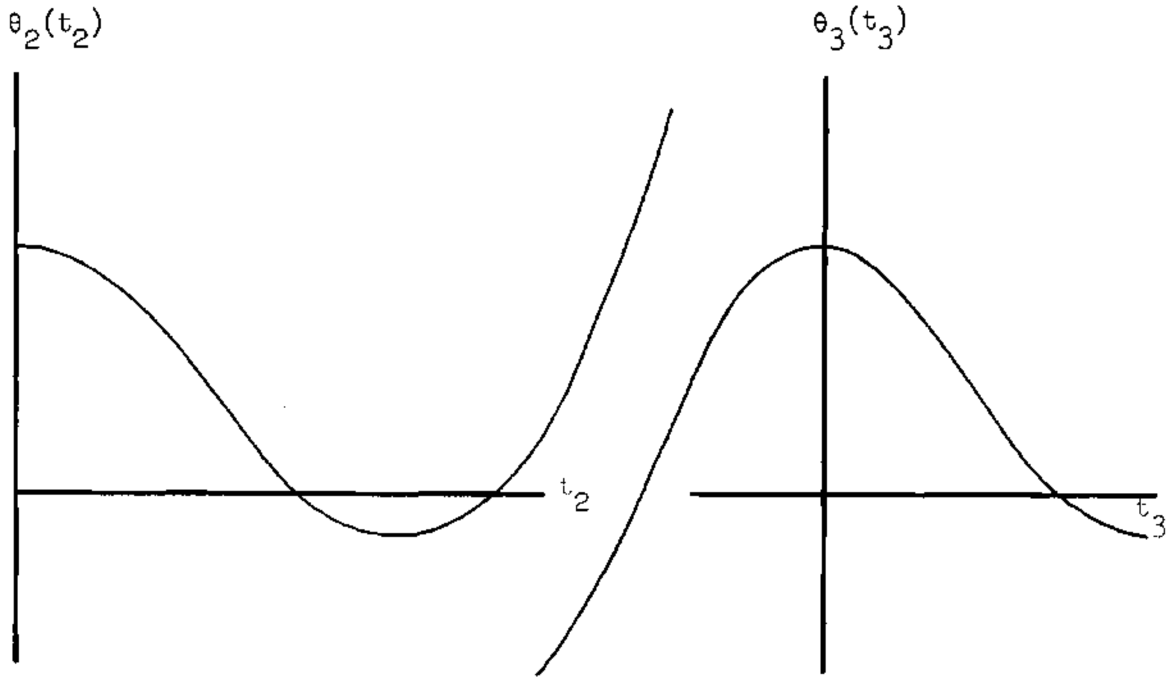


Figure 23. Notation Utilized in the Newton-Raphson Solution.

worth noting that there is a difficulty peculiar to this particular method, and that is its sensitivity to the initial approximation. The natural role of the Newton iteration, according to Scheid (2), is the conversion of a reasonable first approximation into an excellent approximation. There are other algorithms better suited to finding roots when little knowledge of their location exists. Occasionally, given inadequate first approximations, the Newton algorithm will converge to the wrong roots. Therefore, if extensive changes are ever made to the system parameters it would be worthwhile to check the initial approximations used throughout the program for accuracy.

Satisfying the Required Period - (Loop B)

At this stage, the condition requiring matched clutch engagement and disengagement points which result in a smooth, continuous motion has been satisfied. However, there remains an additional constraint - the motion must have the prescribed period τ. Fortunately, there is an additional parameter which may be varied, namely Tr.

In the preceding graphical and numerical solutions it was necessary to pick a value for Tr. This particular choice resulted in a continuous, periodic curve with a particular period. In fact, the calculated period would be:

$$\tau_{calc} = t_2 + |t_3|$$

It is reasonable to expect that for a varying Tr, then, a corresponding variance in τ_{calc} would result. Graphically this might appear as shown below:

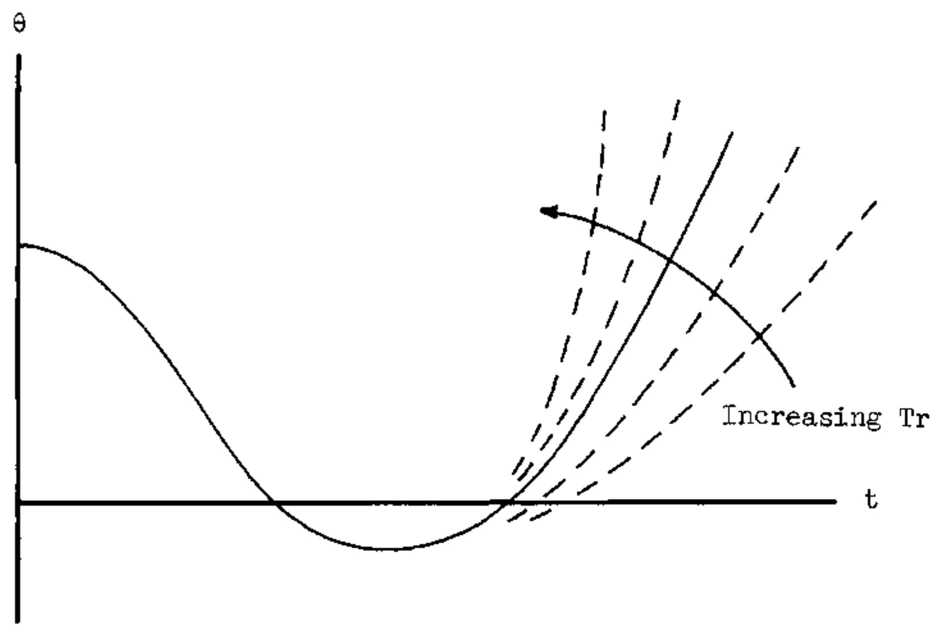


Figure 24. Family of Curves for Varying Tr.

This correspondence satisfies intuition in that a larger slope for the input torque means the torque build-up is faster and in turn produces a faster rate of change in the angular displacement. It is evident, then, from Figure 22 that τ_{calc} decreases for increasing Tr.

The additional constraint of the specified period may now be satisfied with an additional iterative loop which finds the appropriate Tr surrounding the previous one. When both iterations are complete, the load side motion satisfies all specifications. The inner loop and the outer loop are assigned the names loop C and loop B, respectively.

Method of Bisection. The numerical technique used in loop B is known as the method of bisection. If y is a function of x over some interval containing a root of the equation

$$y(x) = 0$$

the method finds successive approximations to the root as follows.

First, the interval containing the root is bracketed by values of x chosen so that one is too large, x_L, and the other is too small, x_s. See Figure 25. The equation for calculating each approximation is

$$x_n = \frac{x_s + x_L}{2} ,$$

the mathematical average of the bracketing values. Thus, the first approximation would look like the curve in Figure 26.

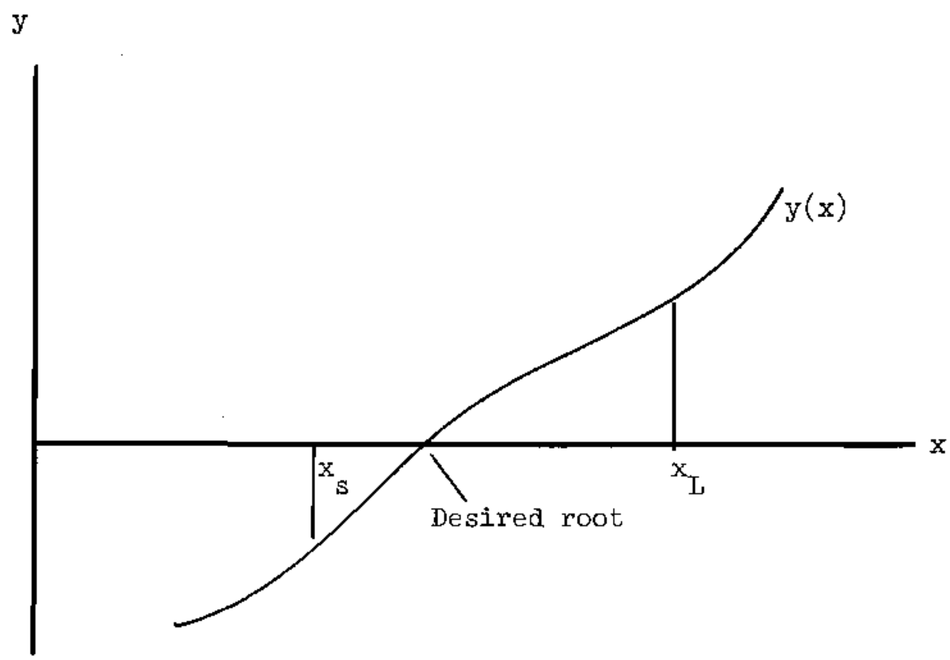


Figure 25. Bracketing the Root.

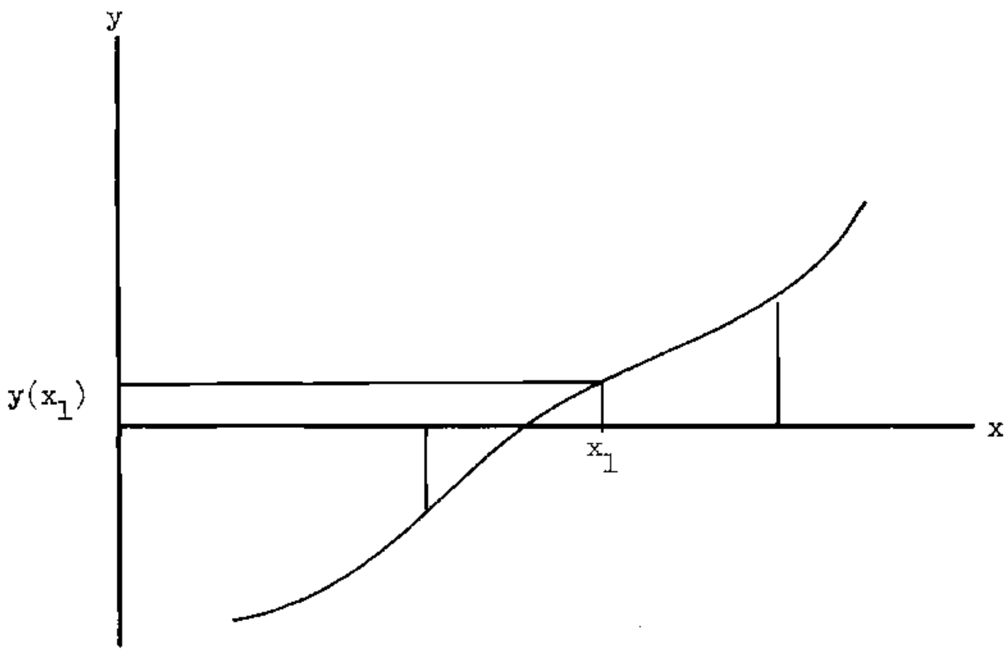


Figure 26. The First Approximation.

Then, y is evaluated at x_n and its sign examined. If $y(x_n)$ is positive it is on the same side of the root as x_L and replaces x_L in the next iteration. If $y(x_n)$ is negative, it is on the same side of the root as x_s and replaces x_s in the next iteration. This order of replacement requires that $y'(x)$ be positive at the root. If the derivative is negative at the root, then if $y(x_n)$ is positive, it is on the same side of the root as $\underline{x_s}$ and replaces x_s in the next iteration, etc.

Therefore, for the example shown, Figure 27, x becomes x_L and the iteration is performed again. The next approximation, x_2 would then appear as below:

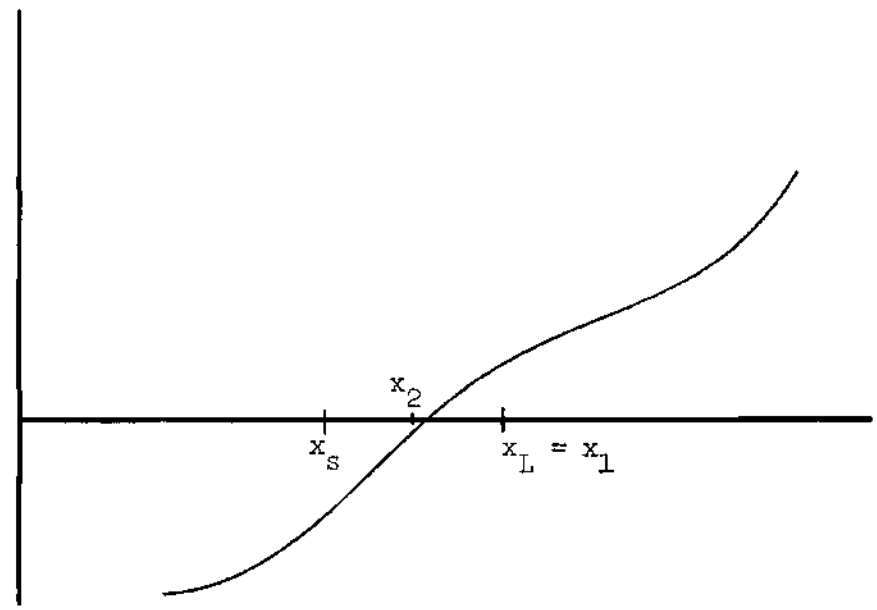


Figure 27. The Second Approximation.

Since $Y(x_2)$ is now negative, x_2 becomes x_s for the third iteration and so on.

The rate of convergence for the method of bisection is not nearly so rapid as for the Newton-Raphson method. However, the useful advantage of this method lies in the freedom allowed in picking x_s and x_L. It can be seen that very little knowledge of the location of the root is required since the bracketing interval can be very large.

Applying the Method of Bisection. This sequence can be set up to find the appropriate Tr. If y(x) is replaced by

$$y(Tr) = \tau - \tau_{calc} = 0$$

with $\tau_{calc} = f(Tr)$, the method of bisection will converge to Tr such that $\tau_{calc} = \tau$. This is illustrated in Figure 28.

The load side motion is satisfied, then, when both loops are complete. The result of the two iterations is a value for Tr and a value for t_2. It is now necessary to carry this information to the other side of the clutch and satisfy the drive side requirements.

Drive Side - (Loops D and E)

The drive side solution seeks the steady-state values of ω_1 and ω_2 for a particular gear reduction ratio. The ratio itself is also determined by a criterion derived from heat dissipative characteristics of the clutch.

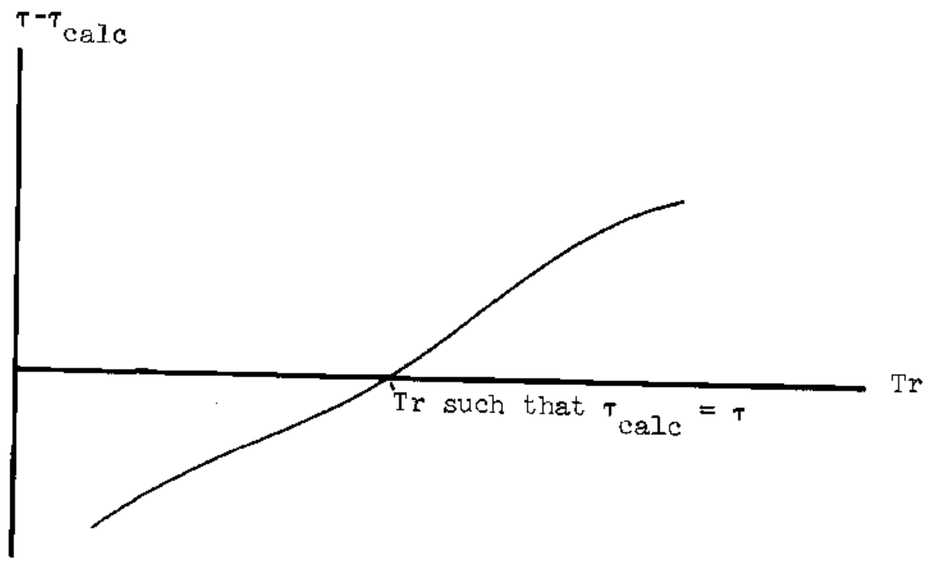


Figure 28. Bisection Applied to Period Requirement.

Equilibrium Values of ω_1 and ω_2 - Loop E

As discussed previously, the steady-state values of ω_1 and, therefore, ω_2 are fixed by the load side solution, i.e., a certain Tr acting over a certain time interval periodically. Therefore the drive side iterations of necessity follow loops B and C.

This problem was solved using the Newton-Raphson method as it applies to a system of one equation in one independent variable. The iteration scheme then reduces to the following formula. For

$$F(x) = 0$$

successive approximations to the root x are given by

$$x_n = x_{n-1} - \frac{F(x_{n-1})}{F'(x_{n-1})}$$

The sequence will converge to the correct ω_1 if $F(\omega_1)$ is chosen by the following reasoning:

$$\text{steady state} \\ \begin{matrix}\text{speed decrease during} \\ \text{engagement}\end{matrix} = \begin{matrix}\text{speed increase during} \\ \text{disengagement}\end{matrix}$$

Mathematically, the appropriate equation is

$$F(\omega_1) = \{e^{\frac{\alpha}{J_F} t_{accel}} [\frac{TQ}{\alpha} + \omega_2] - \frac{TQ}{\alpha}\} - \omega_1$$

where

$$\omega_2 = e^{\frac{\alpha}{J_F} t_{trans}} [\omega_1 + \frac{TQ}{\alpha} - \frac{TrJ_F}{R\alpha^2}] + \frac{Tr}{R\alpha} t_{trans} + [\frac{TrJ_F}{R\alpha^2} - \frac{TQ}{\alpha}]$$

and

$$t_{trans} = t_2 - t_1$$

$$t_{accel} = \tau - t_{trans}.$$

Pictorially the above equation results in the proper juxtaposition of Figures 16 and 17. The N-R routine, then, yields a value for ω_1 for steady state, and simultaneously produces the accompanying value for ω_2.

Controlling Relative Slip in the Clutch - Loop D

The purpose of the gear train may be argued in several ways (see Chapter V) but the mathematics resulting from its inclusion are the same for all viewpoints. The particular criterion utilized for convergence in the computer program, then, is to achieve the lowest possible relative slip in the clutch. This viewpoint strives to minimize the energy lost in the form of heat due to friction between the slipping surfaces.

The relative slip varies in a non-linear fashion throughout the engagement interval. However, the general trend indicates a decrease in slip as engagement continues until some minimum value occurs at or near disengagement. The following routine is based on the fact that if this minimum value is made to approach zero, the energy lost will be minimized accordingly. In other words, the method strives toward having the load just come up to motor speed at disengagement. In reality there are certain restrictions on this technique which will be discussed in Chapter V.

The criterion used for determining R, then, is as follows. R is to be selected so that the motor side speed of the clutch equals the load side speed times a factor at disengagement. (This factor is explained fully in Chapter V, its inclusion having evolved from early results). Mathematically this is expressible as

$$\frac{\omega_2}{R} = \dot{\theta}_{II}(t_2) \cdot \text{Factor} \tag{13}$$

where

$\dot{\theta}_{II}$ is the load velocity during stage II, and

ω_2 is by definition motor speed at t_2.

It can be seen that if Factor equals 1.0 then the motor side and the load side of the clutch will arrive at exactly the same speed at disengagement.

Equation (13) may be rearranged to fit the method of bisection. If y(x) on page 55 is replaced by

$$y(R) = \dot{\theta}_{II}(t_2) \cdot \text{Factor} - \frac{\omega_2}{R} \tag{14}$$

then the method will converge to R such that the right hand side is zero.

The drive side motion is satisfied, then, when both loops D and E are complete. The motor then cycles between two steady-state values while minimizing energy expended in heating the clutch.

Total System - Loop A

The net result of loops B through E is that the total system is in equilibrium and will operate in a smooth continuous fashion ad infinitum. However, of immediate concern is the horsepower being extracted from the motor. The "smooth" motion above may be badly overloading the motor if not possibly stalling it (since the equations have no way of sensing breakdown torque).

A final "loop" is included to examine the power aspect. The term "loop" is used loosely here for the actual process consists of a READ statement in the program. The loop is closed by the programmer who chooses the succeeding approximations and enters them in the form of data.

The remaining parameter which may be varied is the load spring constant k. The initial value had been chosen so that the damped natural period approximated the desired period. A variation in k effects the amount of energy the spring can store for a given displacement, and thus alters the interplay between kinetic and potential energy for the system. It is reasonable to assume that by submitting as data various values for k, a corresponding variation in motor energy per cycle may result.

Thus, the computer program in its final form consists of five nestled iterations. A pictorial version showing the relationship of the various loops appears in Figure 29. A more detailed flow chart as well as the actual computer program in FORTRAN IV language is included in Appendix A for reference.

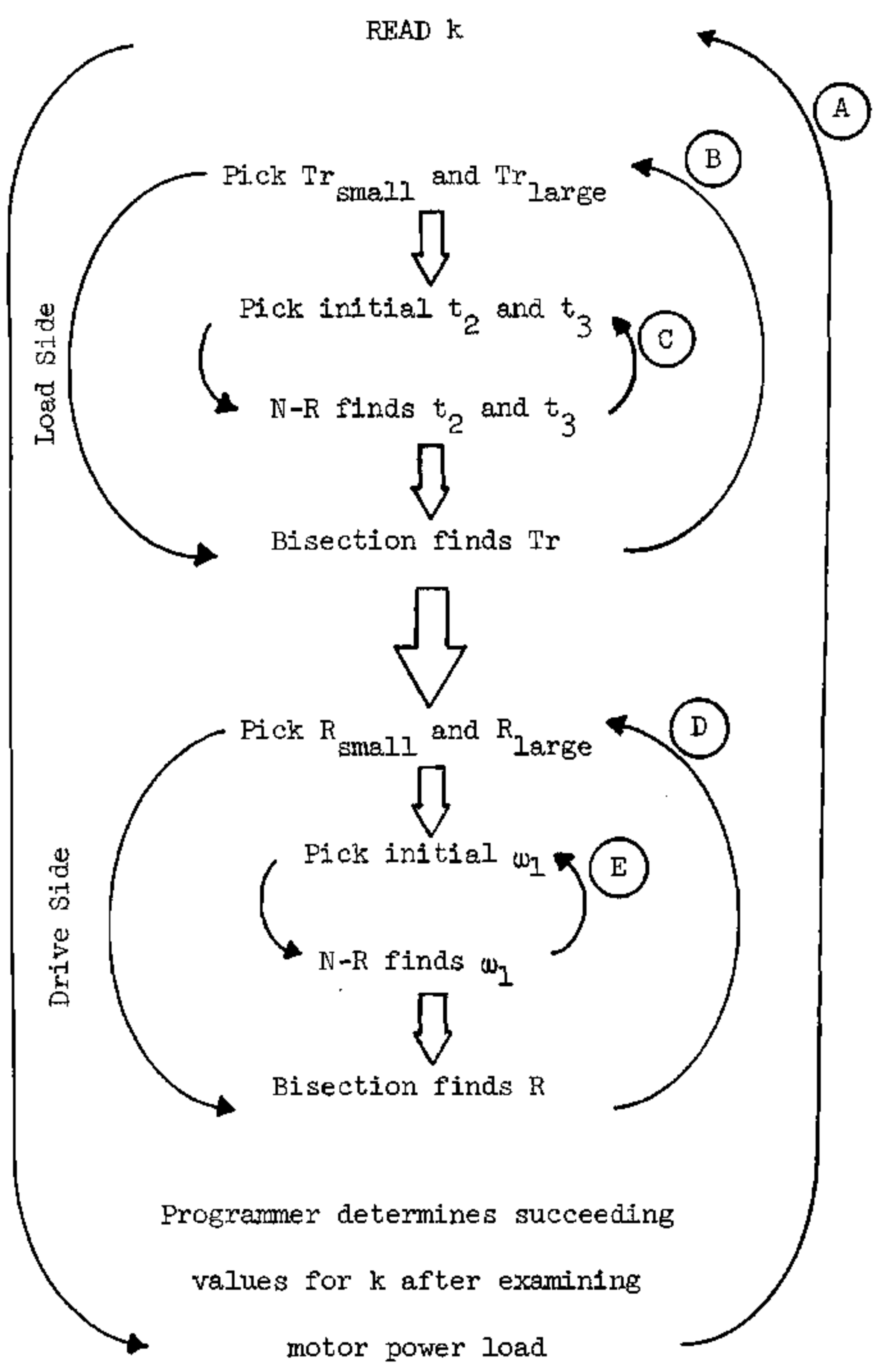


Figure 29. Nestled Loop Configuration.

CHAPTER IV

CLUTCH HEATING AND SYSTEM STABILITY

That the system will perform adequately is not merely a matter of satisfying the desired motion requirements. Any mechanism which is built to serve a useful purpose must, in general, meet two more criteria. The machine should have a reasonable life expectancy in terms of projected workload, and it should be able to withstand expected deviations from the design conditions. The first criterion usually requires specification of a design life, while the latter generally takes the form of a service factor of some sort.

In view of the feasibility nature of this study, the life question was approached in the following manner. No attempt was made to specify the actual physical design of components such as shafting and gears (as this would be the next step in the design procedure), however components of this type may always be made rugged enough. In this early stage of analysis the most beneficial approach is to examine only those areas of a complex machine where problems might be forecast. For this particular mechanism these were deemed to be clutch wear, clutch heating and motor heating. All of these are discussed in Chapter V, however the mathematical derivations for the clutch temperature rise are presented here.

The second criterion is not satisfied by a service factor in this case. The problem essentially consists of determining whether or not the system is stable under varying inertias and/or damping. A stable system is expected to be self-adjusting, that is, if a design parameter is varied, the system will seek a new steady-state operating mode. On the other hand, an unstable system would be characterized by load displacement and velocity growing without bound or possibly diminishing completely. Again, only the mathematical basis is presented here with discussion of results included in Chapter V.

Clutch Heating

The various factors affecting temperature rise in a clutch are listed as follows:

(1) Amount of heat generated each clutching operation.

(2) Relative time spent engaged and disengaged each cycle.

(3) Cycling rate.

(4) Heat transfer to surroundings.

The relative times and cycling rate are obtained as solutions from the previous computer program. Factors (1) and (4) are obtained as follows.

Heat Generated Each Clutching Operation

If an energy balance is performed on a control volume surrounding the clutch as in Figure 30, the following is true for steady motion over a cycle.

$$W_{in} = W_{out} + Q_{dissipated}$$

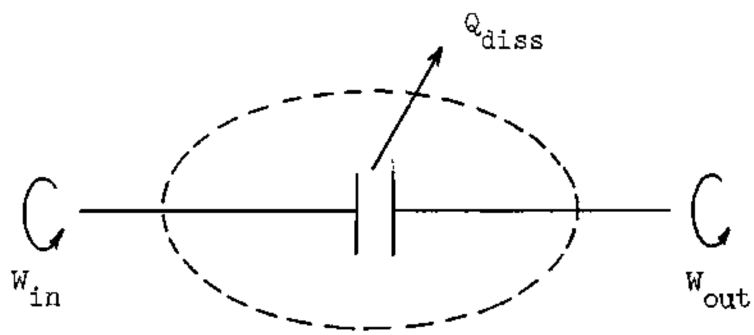


Figure 30. Clutch Control Volume

or

$$Q_{diss} = W_{in} - W_{out}$$

$$Q_{diss} = \int_{t_1}^{t_2} T_{in}\, \dot{\theta}_{in}\, dt - \int_{t_1}^{t_2} T_{out}\, \dot{\theta}_{out}\, dt$$

$$= \int_{t_1}^{t_2} [T_{in}\, \dot{\theta}_{in} - T_{out}\, \dot{\theta}_{out}]\, dt$$

and since $T_{in} = T_{out} = T(t)$,

$$Q_{diss} = \int_{t_1}^{t_2} T(t)\ [\dot{\theta}_{in} - \dot{\theta}_{out}]\ dt \tag{15}$$

Thus there are three controlling elements - interval of engagement, transmitted torque, and relative velocities of input and output. The torque and transmission interval may be affected by k, the spring constant, but much more accurate control may be obtained by controlling the slip with the reduction ratio R. Actually both methods are used and they appear in the program as loops A and D respectively.

Some insight is gained into the slip situation by the following. Though an extension of Newton's third law reveals that the torque build-up has to be the same for both input and output, the speed variation for input and output may differ considerably. For the particular load system under consideration the situation appears as shown in Figure 31. The slip, represented by the difference in the upper and lower curves at (c), then, must be minimized over the interval to minimize the heat loss.

A numerical integration technique is utilized to calculate Q after the entire procedure of Chapter III is complete. That is, after k, R, T_r, t_1 and t_2 are known. The technique utilized is known as the trapezoidal rule and as the name indicates, it sums the areas of trapezoids fitted under the curve. Thus, the curve is approximated

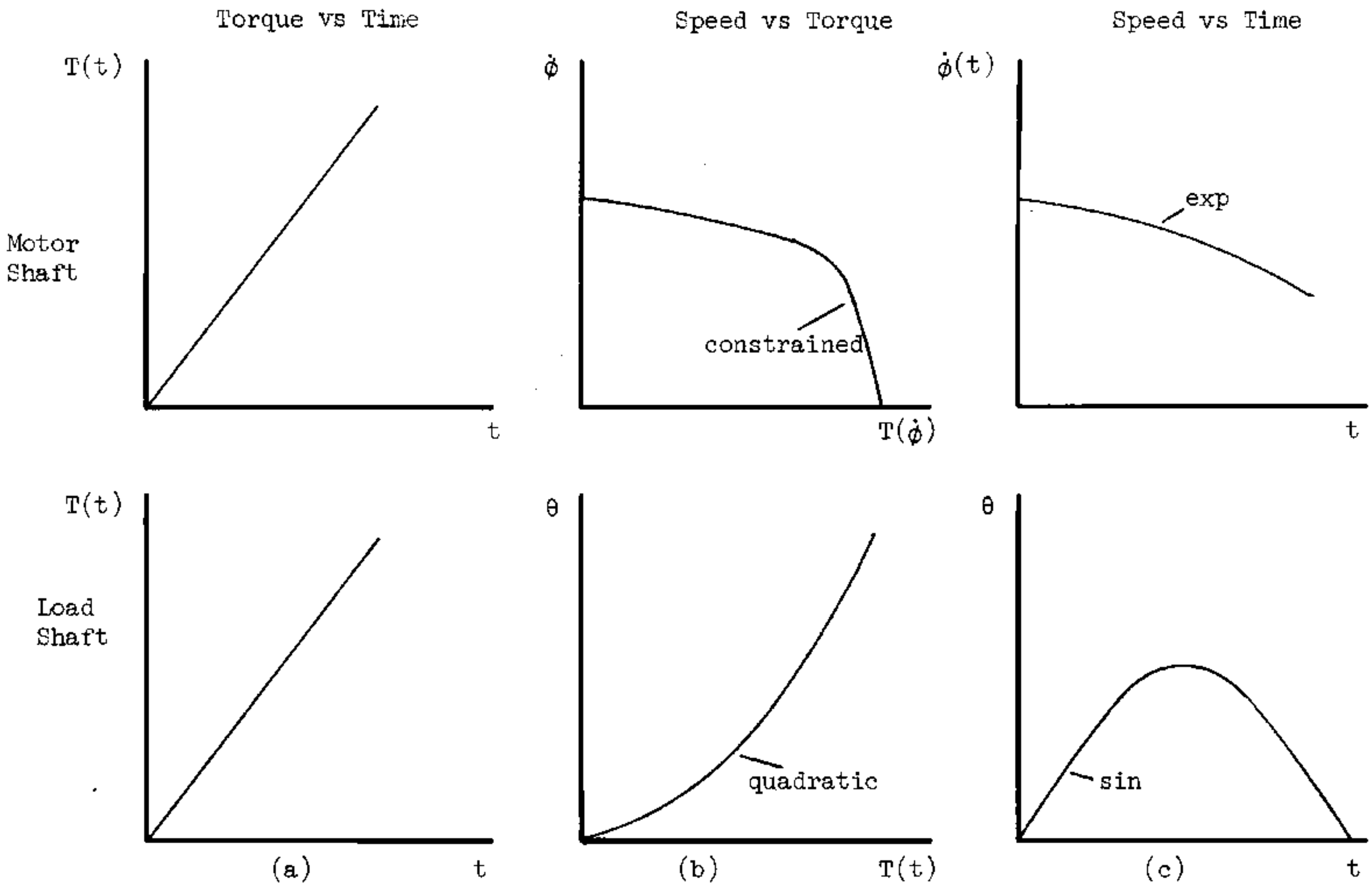


Figure 31. Clutch Slip

by a series of straight line segments. The composite formula is given below.

$$\int_{x_o}^{x_n} y(x)dx \sim \tfrac{1}{2}h\ [y_o + 2y_1 + \ . \ . \ . + 2y_{n-1} + y_n]$$

where $h = \Delta x$ = width of trapezoid. Thus, if

$$y(t) = \mathrm{Trt}\ [\dot{\theta}_{in}(t) - \dot{\theta}_{out}(t)\ ]$$

equation (15) is approximated by

$$Q_{diss} = \int_{t_1}^{t_2} y(t)dt \sim \tfrac{1}{2}h\ [y_o + 2y_1 + \ . \ . \ . + 2y_{n-1} + y_n] \qquad (16)$$

and h = incremental division of $[t_1 t_2]$. As in all numerical integration techniques, there are various sources of error present in equation (16). However, according to Scheid (2), the wide variety of efforts to estimate this error are somewhat unsatisfactory. The difficulty arises if the integrand is anything other than a trivial function.

According to Shigley (3), any effort to compute clutch heating must be extremely rough anyway due to inability to accurately assess the rate of heat transfer. A calculation will indicate, at best, whether or not the temperature rise is likely to be a major consideration. Therefore, based on this reasoning, an error analysis was not attempted.

Heat Transfer Coefficient

The heat generated is dissipated by radiation, convection, and conduction. With all three modes of heat transfer working simultaneously, it is convenient to specify an "overall" heat transfer coefficient. The coefficient must take into consideration the character of the surroundings, the ventilation, the nature and shape of the radiating and conducting surfaces, and the temperature distribution throughout the clutch. Thus most researchers surveyed rely on experimentally determined values. Gagne (4) suggests values between 1.5 and 5.10 Btu/hr-ft^2-$^\circ$F for surface velocities ranging from 0 to 100 fps. Fazekaz (5) offers 2.3 - 4.8 over a similar range.

Solutions

A literature survey revealed three categories of approaches to the problem of temperature rise. The first group attempts to solve the theoretical problem by use of Fourier equations and considers this kind of heating as a normal case of heat transfer. The papers in the second group aim at solving the problem by employing invariants and similarity laws. The last group comprises experimental work done on temperature measurement and analysis of the data obtained.

Representative equations from the first and last groups were judged to be most in line with the particular problem at hand. The theoretical (or experimental) bases for these results, which are too lengthy to present here, may be obtained via the List of References. The solutions of Fazekaz (Newtonian law of cooling), and Gagne (empirical) respectively are as follows:

$$T''_{\infty} = T_o + \frac{Q}{hAt_o}\left(bt_o + \frac{bt_o}{e^{bt_o} - 1}\right), \; b = \frac{Ah}{Wc} \tag{17}$$

$$t_{av} = \frac{HN}{AC} \frac{1}{\left[\frac{Nt}{3600} + 1.5\left(1 - \frac{Nt}{3600}\right) \right]} + t_1 \tag{18}$$

with notation

T''_{∞} = ultimate clutch temperature (^oF)

t_{av} = average clutch temperature (^oF)

$T_o = t_1$ = initial (ambient) temperature (^oF)

$Q = H$ = heat generated each engagement (Btu)

$h = C$ = heat transfer coefficient (Btu/sec-ft^2-^oF)

A = exposed surface (ft^2)

t_o = period of cycle (sec)

W = mass of clutch parts (lbm)

c = specific heat (Btu/lbm-^oF)

N = number of cycles per hr.

t = time of single clutching operation (sec)

The equations are seen to be very similar and both papers indicate that the temperature will climb in a saw tooth fashion until an equilibrium condition is obtained, Figure 32. Note that the temperature cycles with clutch application even for large values of time.

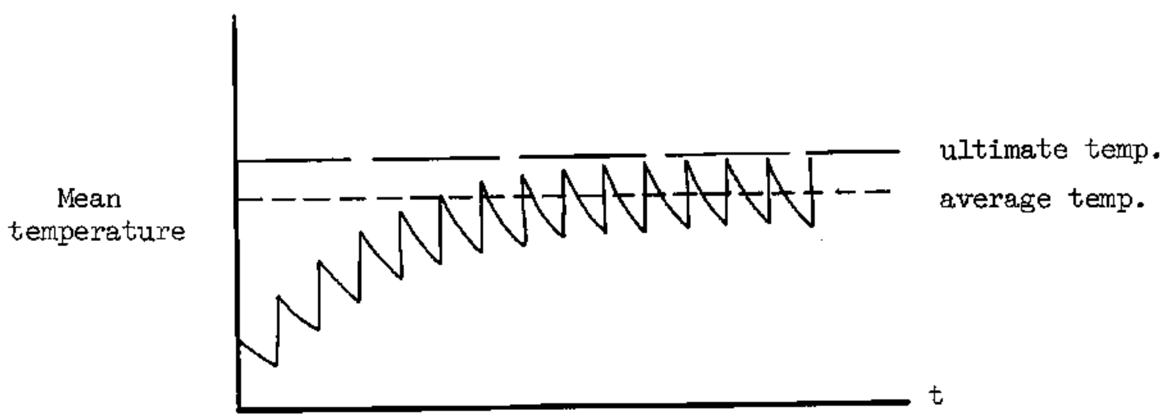


Figure 32. Clutch Temperature Rise

Stability

The program outlined in Chapter III serves its only function in obtaining solutions for certain design parameters. If the damping is changed, new solutions for T_r, t_1, t_2, etc. will result such that the desired motion is maintained. This is to say that if the output motion is required to remain constant for any loading and damping combination, then a computer must be a functional component of each machine. Of course, a more reasonable approach would be to allow moderate deviations from the specified motion. Therefore, the behavior of a realistic machine under varying conditions was investigated.

For instance, it seems likely that the most practical method of signaling the proper clutch engagement and disengagement times would be by angular position of the load. This is as opposed to the main program which performs all sensing and signaling in terms of time. But as is evident from Figure 33, t_1 and t_2 each have a corresponding angular position.

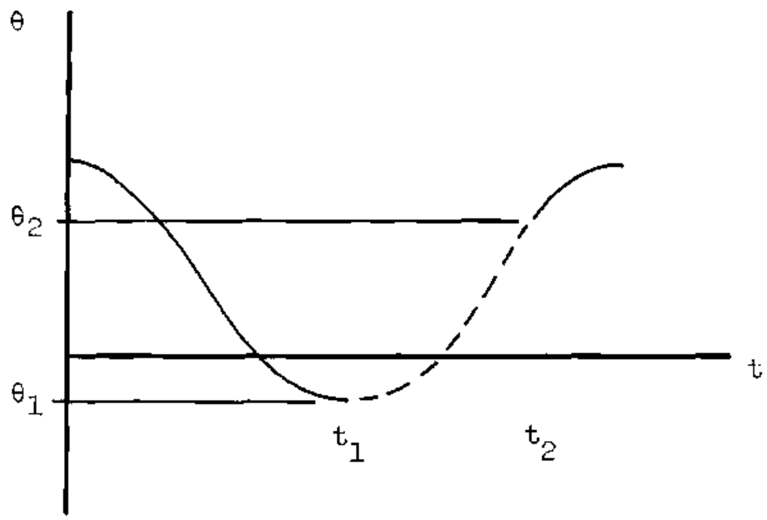


Figure 33. Positions θ_1 and θ_2

From this, then, a reasonable design approach might be the following. First, the worst possible conditions are envisioned, i.e., heaviest load damping. Then utilizing the main program obtain T_r, t_1 and t_2. Also determine the corresponding positions θ_1 and θ_2. Next, via linkages or electrical switches construct the actuator mechanism so that engagement occurs at θ_1 and disengagement occurs at θ_2. Again, it should be restated that this appears to be the simplest approach as opposed to sensing time or velocity. The mechanism would now operate exactly as designed producing the desired output for the design parameters.

If, then, the tub load is altered, causing a corresponding alteration in the damping, a new response might logically be expected. Since the design value was a maximum, any change must amount to a reduction in damping. The first sequence of events can be predicted as follows.

The load would accelerate more rapidly from the initial position and would pass θ_1 at some time less than t_1. The clutch would be engaged and would act to retard the excessive swing due to the clutch discs having opposite velocities. The negative velocity would stop and the load would begin accelerating with positive velocity toward θ_2. At θ_2 the torque assist would disengage (t is considerably less than t_2) and from this point on the motion is not predictable. The plot in Figure 34 graphically portrays the above sequence.

The solution involved writing a new program to simulate a real machine. The problem essentially reduces to computing when to shift equations to simulate clutch action. Thus the sensing of where (position) must be translated into when (time).

For this purpose a set of function subprograms named WHEN1, WHEN2, and WHEN3 were devised. These subprograms follow a numerical

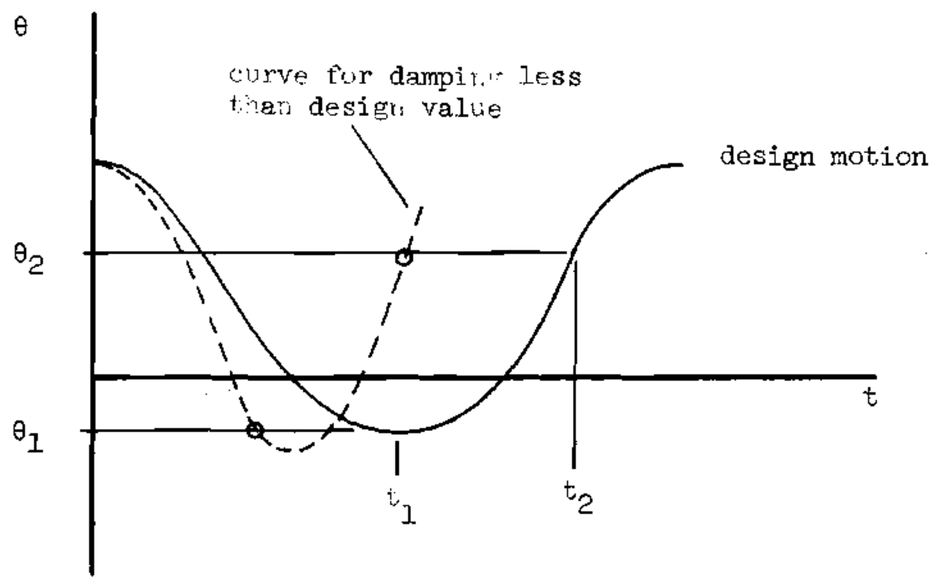


Figure 34. Possible Motion for Damping Other than Design Value

procedure which finds the times when particular positions are reached. For instance, WHEN1 seeks out the time when θ_1 is reached and assigns this value to $(t_1)_i$. Thus, t_1 and t_2 become subscripted variables since they may vary. The program allows for new values for t_1 and t_2 each cycle. WHEN3 performs a similar function necessary in calculating an average period, etc. since all parameters may change each cycle.

The stability program offers considerable flexibility and therefore is a useful design tool. For instance, if it was deemed simpler or more practical to signal clutch actuation by velocity, only the three subprograms would need alteration. The same format, even then, could be utilized and the new subprograms written relatively painlessly.

Also, since the stability program behaves like a real mechanism, other interesting tests may be made. Realizing that it would be much more convenient to start the motion from the initial conditions

$$\theta(0) = 0$$

$$\dot{\theta}(0) = 0$$

it is desirable to investigate this possibility. The results obtained from these various investigations are contained in Chapter V. The stability program itself is reproduced in Appendix B for reference.

CHAPTER V

ANALYSIS OF RESULTS

Foreword

The purpose of this research, being analytical in nature, is to serve as the first step in implementing a proposed mechanical design. The proposed mechanism, having been modeled mathematically, is analyzed for features which might determine its feasibility on a physical scale. Some degree of optimization is pursued, however the basic aim is to establish general guidelines and ranges for system parameters such that a physical model may be constructed.

The results discussed in this chapter stem from the successes and failures as compiled from computer output. In this capacity the computer serves as the principle tool in generating data to be analyzed. To facilitate correlation with the developmental chapters, the discussion follows the format of load side, drive side, and finally the total system. Lastly, on the basis of conclusions reached in the analysis, recommendations are made for future research.

Discussion and Conclusions

Load Parameters

Typical load design parameters as obtained from the Whirlpool Corporation are as follows:

$$J_T \simeq 4.44 \text{ in-lbf-sec}^2$$

$$c \simeq 28.8 \text{ in-lbf-sec}$$

These represent experimentally-determined values, and, as such, are approximate. The damping due to fluid sloshing is complex and varies throughout a cycle making a measured value have the form of an equivalent or averaged damping.

Other design parameters which are specified by the designer are the desired arc of motion and the period. Again, typical values are

$$\text{Arc} = 270^\circ = 4.7124 \text{ rad}$$

$$\tau = 1.4 \text{ seconds}$$

These values comprise a portion of the input data to be read into the computer. There are also certain fixed parameters associated with the drive side.

Drive Parameters

The drive side has essentially three components - motor, flywheel, and gear train. The fixed parameters associated with the motor were obtained for a typical fractional horsepower unit. Preliminary estimates indicated a power requirement of approximately one half horsepower so a motor of this rating was utilized. The torque-speed characteristics yield

$$\alpha = -1.5 \quad \text{in-lbf-sec/rad}$$

$$TQ = 280 \quad \text{in-lbf}$$

$$\text{Breakdown speed} = 1600 \text{ rpm} = 168 \text{ rad/sec}$$

The flywheel inertia was chosen so as to best represent a practical choice based on weight and physical size limitations.

$$J = 1.0 \quad \text{in-lbf-sec}^2$$

It is helpful to compare this value with the load inertia noting a factor of approximately four between them. Also, an intuitive grasp of these values is obtained by the following:

For a solid disc

$$J = \frac{1}{2}\left(\frac{w}{g}\right)r^2 \quad \text{or} \quad w = \frac{2Jg}{r^2}$$

Therefore, for $J = 1.0$, $r = 6$ in., the disc weighs

$$w = 21.5 \text{ lbf}$$

The drive side fixed parameters are completed with the specification of the clutch slip factor at disengagement:

$$\text{Factor} = 1.05$$

Discussion of this value is postponed until later in this section.

Load Side Motion

The order of execution in the program is such that the load side motion is satisfied first. In fact, should there be no solution for the load side, the drive side portion would have no meaning. In view of the constraints on the motion and the fixed function for the external torque there remained the possibility that the proper combination might not exist for the smooth, continuous motion as desired. If this were the case, the proper functional form for the external torque would have to be determined. Fortunately, the ramp torque is entirely satisfactory. However, as the above discussion indicates, it is not necessarily the optimum form for T(t).

The ramp torque function is satisfactory only in that the slope is left as a variable. As soon as the slope is fixed, as in requiring that a particular clutch be used, the constraint of a desired periodicity must be removed. In other words, the system is determined.

It must be pointed out that the resulting motion represents a considerable distortion of a true sinusoid when its derivation is reviewed. Figure 35 traces the steps necessary in achieving a particular motion. Part (a) depicts the desired motion - a true sinusoid of amplitude $\bar{\theta} = 135^{\circ}$ and period $\tau = 1.4$ seconds. The desired swing in each direction is then 270°. The rate of oscillation is 42 strokes per minute with 540° travel per stroke. To obtain this motion via the methods outlined previously, an entirely different system with the following characteristics is needed - a true sinusoid

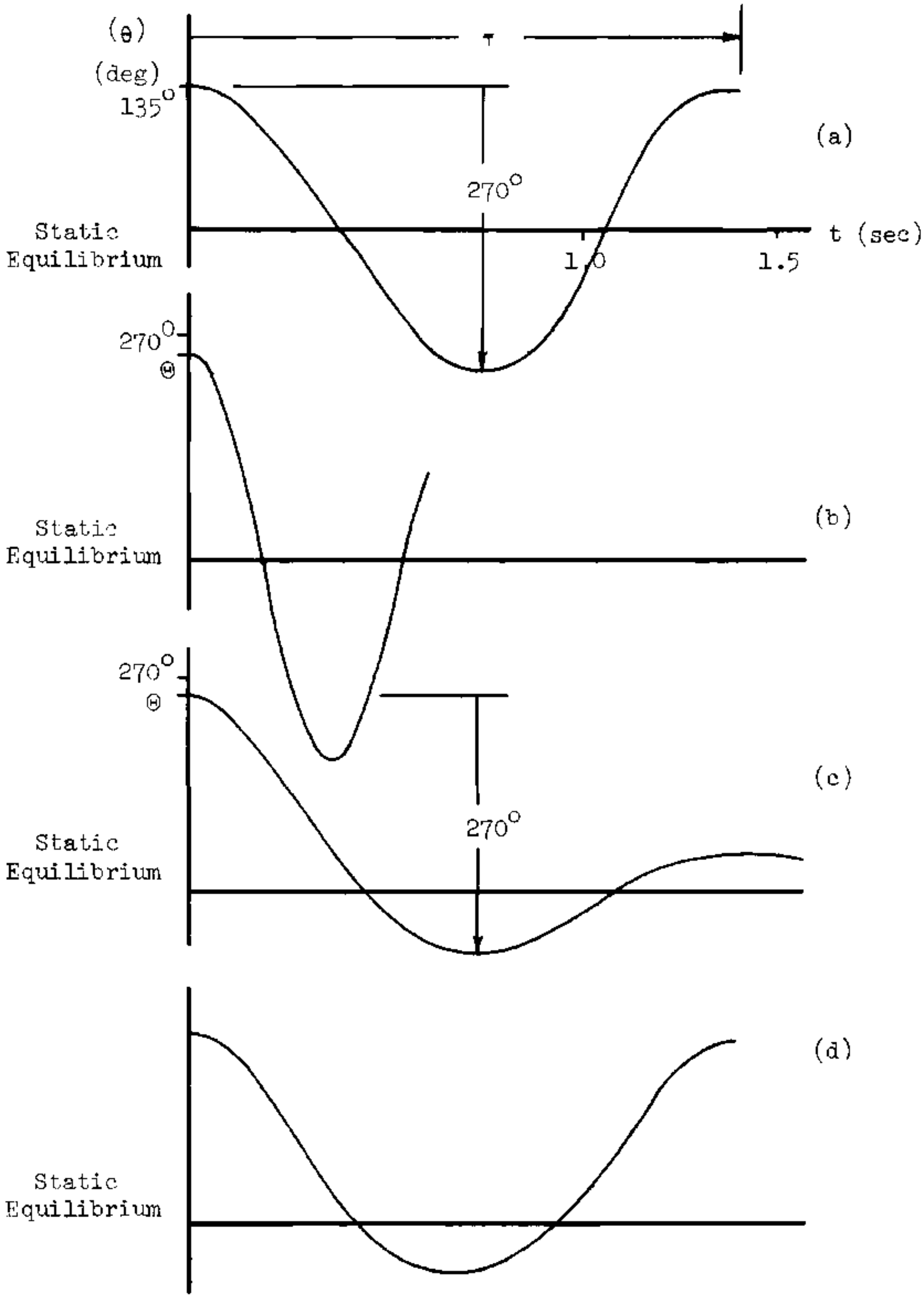


Figure 35. Steps in Achieving the Desired Motion.

of amplitude considerably greater than $\tilde{\theta}$ and period considerably shorter than τ as shown in part (b). When the heavy damping is added, the resulting motion for the half-period is shown in part (c) to be similar to that desired. The motion is completed by applying the proper torque, part (d) resulting in the desired stroke and stroke rate. However, for certain combinations of parameters the sinusoidal appearance becomes noticeably altered. The two extremes are presented in Figure 36 indicating the flexibility of a given system in meeting a range of specified periods.

A sample of actual angular displacement plot is shown in Figure 37. This is a very good approximation to a sinusoid, however, the distortion is made more visible in Figure 38 with expanded time scale.

Once this motion is established, it is independent of alterations made on the drive side as long as the correct $T(t)$ is applied. The spring constant stands as the only parameter which affects the load side. However, variations in this parameter and their purpose are more appropriately discussed with the total system.

<u>Drive Side Motion</u>

While the load side motion is constrained by the fixed parameters of the system, and as such is predetermined, the drive side motion represents a harmonious balance of efficiency and practicality. In addition, certain physical bounds exist which confine useful parameters even further.

The solution is restricted at the outset by the prescribed

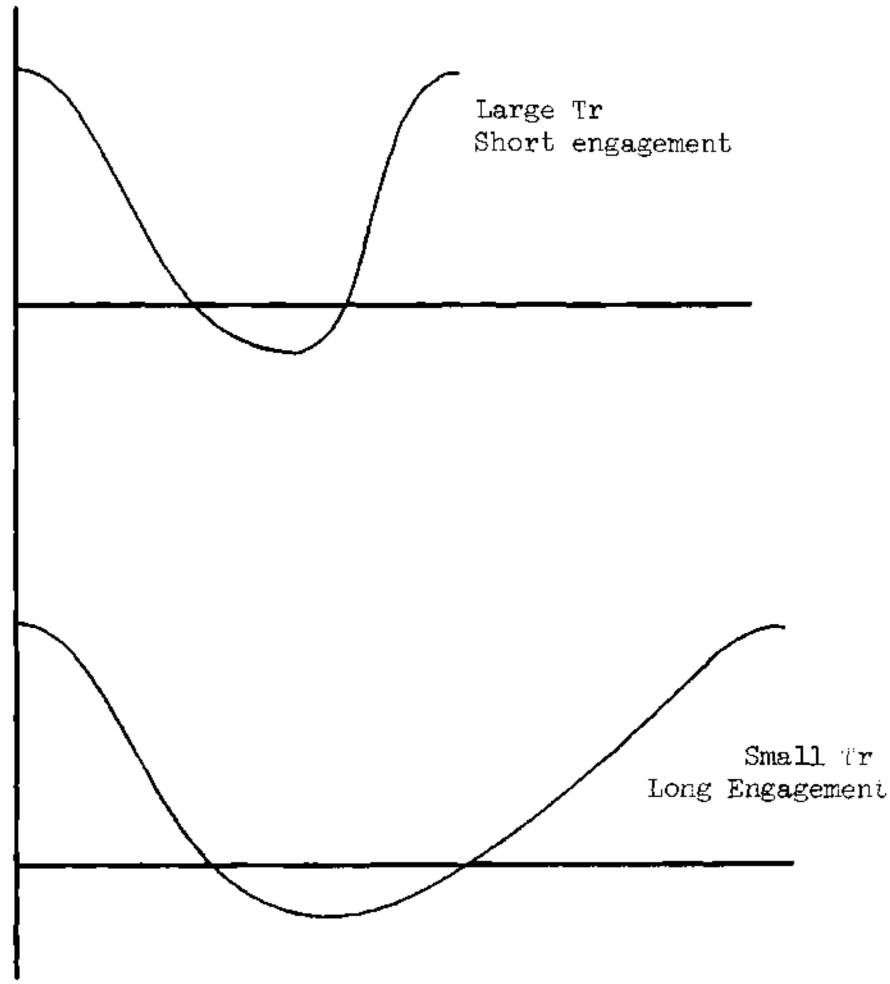


Figure 36. Distortion of Sinusoidal Appearance for Varying Cycle Period.

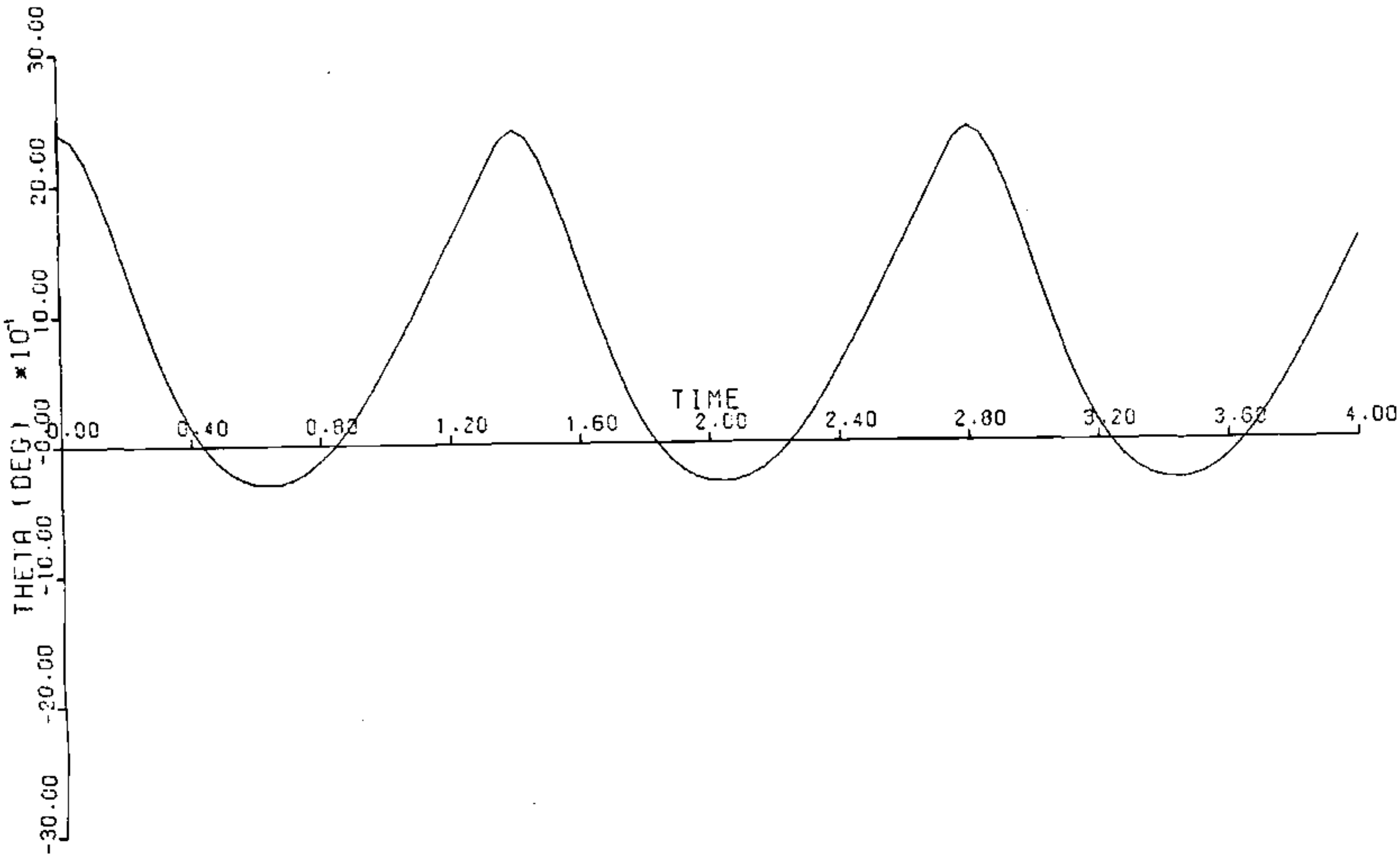


Figure 37. Computer Plot of Tub Angular Displacement Versus Time.

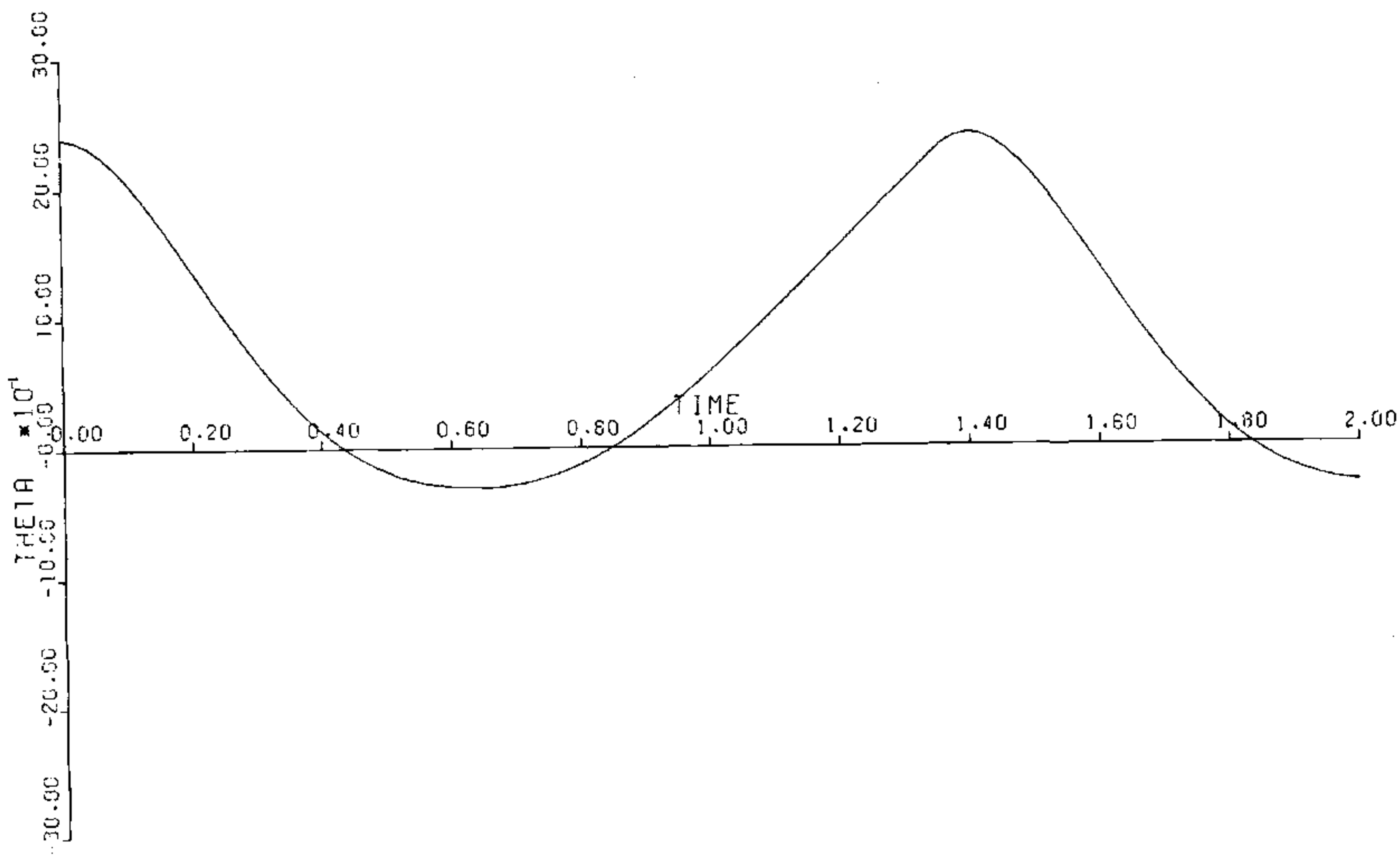


Figure 38. Computer Plot of Tub Angular Displacement
with Expanded Time Scale.

motor and flywheel. There is, of course, considerable leeway in selecting a motor since factors other than performance alone must be considered. In general, the smallest, least costly and most readily available unit which will work (i.e., perform in all the modes of operation for a reasonable design life) is the best choice. Cost and availability being matters somewhat outside this study, the motor selected for use reflects primary concern for satisfactory performance while rating favorably in other categories. The choice of this particular motor, then, will affect solution parameters in that they must satisfy motor performance. Correspondingly, the specification of the flywheel inertia further narrows the range of useful solutions. However these both are necessary restrictions. Motor selection is far too complex to be handled by equations and the optimum flywheel is simply the largest one which is physically reasonable.

Within these constraints, solutions were sought in the manner outlined in Chapter III - at first without gearing. Early results indicated that the motor was undergoing too large a speed fluctuation, thus operating at overload and even past breakdown. The equations dictated greater torque at a lower speed - exactly the effect of a gear reduction. The system chosen is illustrated on page 44 in Chapter II. Several benefits arise out of this system:

(1) The transmitted torque is reflected onto the motor shaft as $T(t)/R$, where R is the step down ratio.

(2) The motor and flywheel are allowed to run at high speed.

(3) The drive side clutch velocity is low, (thus reducing

the heat generation and wear).

Accompanying the inclusion of the gearing is increased complexity and cost along with the new variable R. However, a considerably larger motor is the only substitute.

As with the flywheel, the optimum reduction ratio is the highest practical one. In this case, however, the allowable range of R's is bounded at both ends. Depending on the system, for low ratios (1-18) the motor is still required to operate at overload and for high ratios (16-∞) clutch operation is violated. The physics of clutch transmission requires that the drive side rotate faster than the output side to transmit torque at all. For large reduction ratios the motor side of the clutch turns too slowly. Unfortunately, since the two ranges may overlap for certain systems, there may be no solution. The only remaining parameter variable, k, effects the entire system and is precisely what is needed. Thus, loop A becomes necessary.

Total System

As discussed in Chapter III, a variation in the spring constant has the net effect of altering the interplay of energy between the spring, motor, and flywheel. For an increase in k, the spring delivers a greater portion of the total energy requirement. Thus, the motor workload is reduced. Also, the variation in k alters the load motion slightly. The sum effect of these and other changes throughout the total system is to separate the two problem ranges on the reduction ratio encountered above, thus making a solution possible.

The program is arranged in the following solution procedure. First, a value for k is selected. For each k, the correct value of R is found to minimize the clutch slip rate. If the motor load is still too high, then the correct procedure would be to increase k until it is at an acceptable level.

The clutch slip factor is introduced to allow reducing the slip as far as practicable while avoiding the problem of having the load side of the clutch catch up with the drive side before disengagement. The problem which may be encountered is illustrated in Figure 39 for FACTOR = 1.0. Angular velocity is displayed for both sides of the clutch. It can be seen that although the velocities are forced to match at disengagement by this FACTOR, intersection of the curves prior to t_2 indicates that the load velocity has "caught up" with the motor side velocity ahead of schedule. Thus, if FACTOR is increased to 1.05 as in Figure 40 this problem is alleviated.

The final mechanism, then, reflects the attempt to keep motor speed up, but also to keep clutch relative slip velocity down. Meanwhile, all load side requirements must be met and the load velocity must not catch up to the motor side clutch velocity before disengagement. The program seeks the proper combination of parameters so that the desired load motion is accomplished while the motor enjoys a power loading commensurate with its capabilities. (See Appendix C for additional discussion).

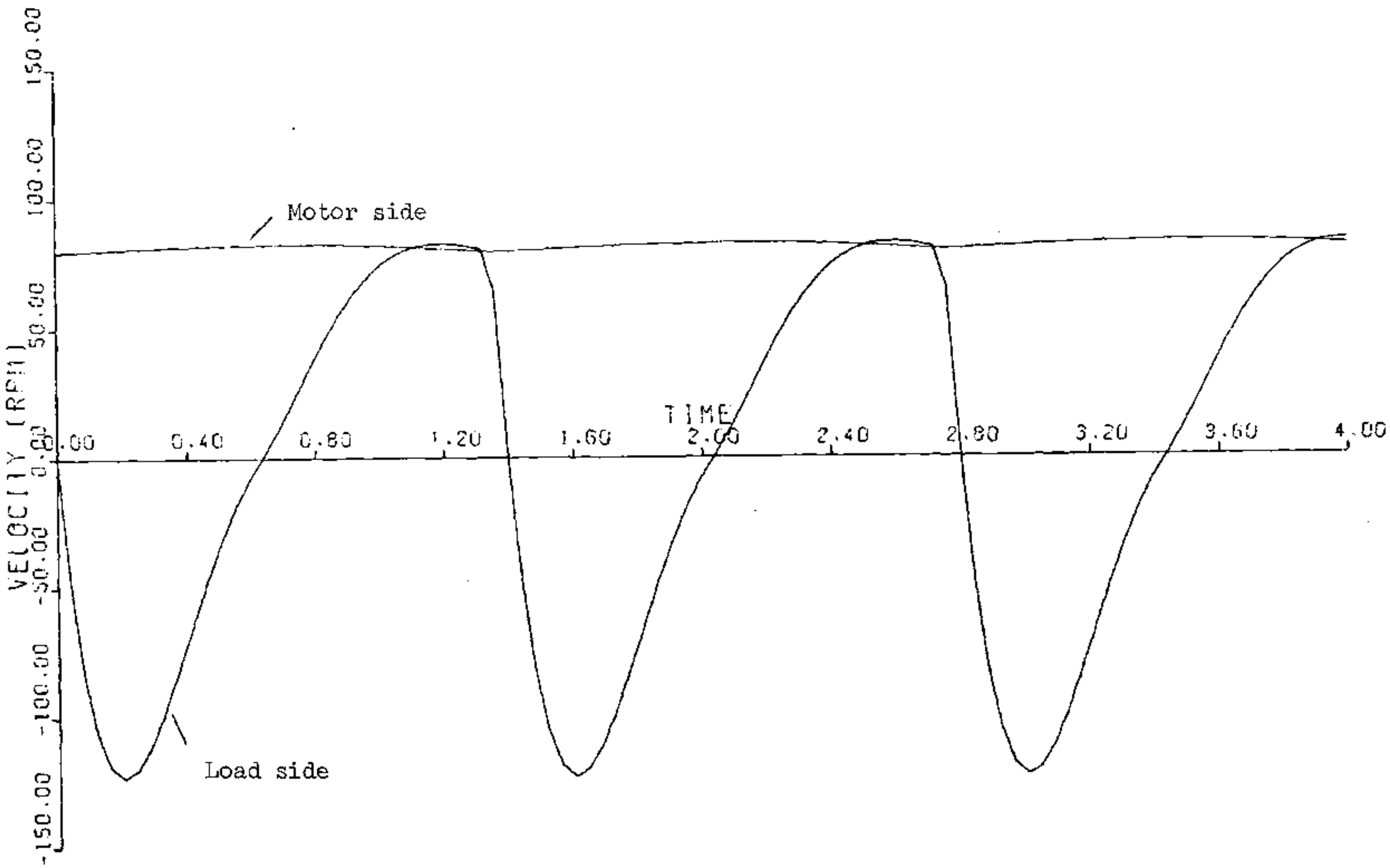


Figure 39. Motor Side and Load Side Clutch Velocities for FACTOR = 1.0.

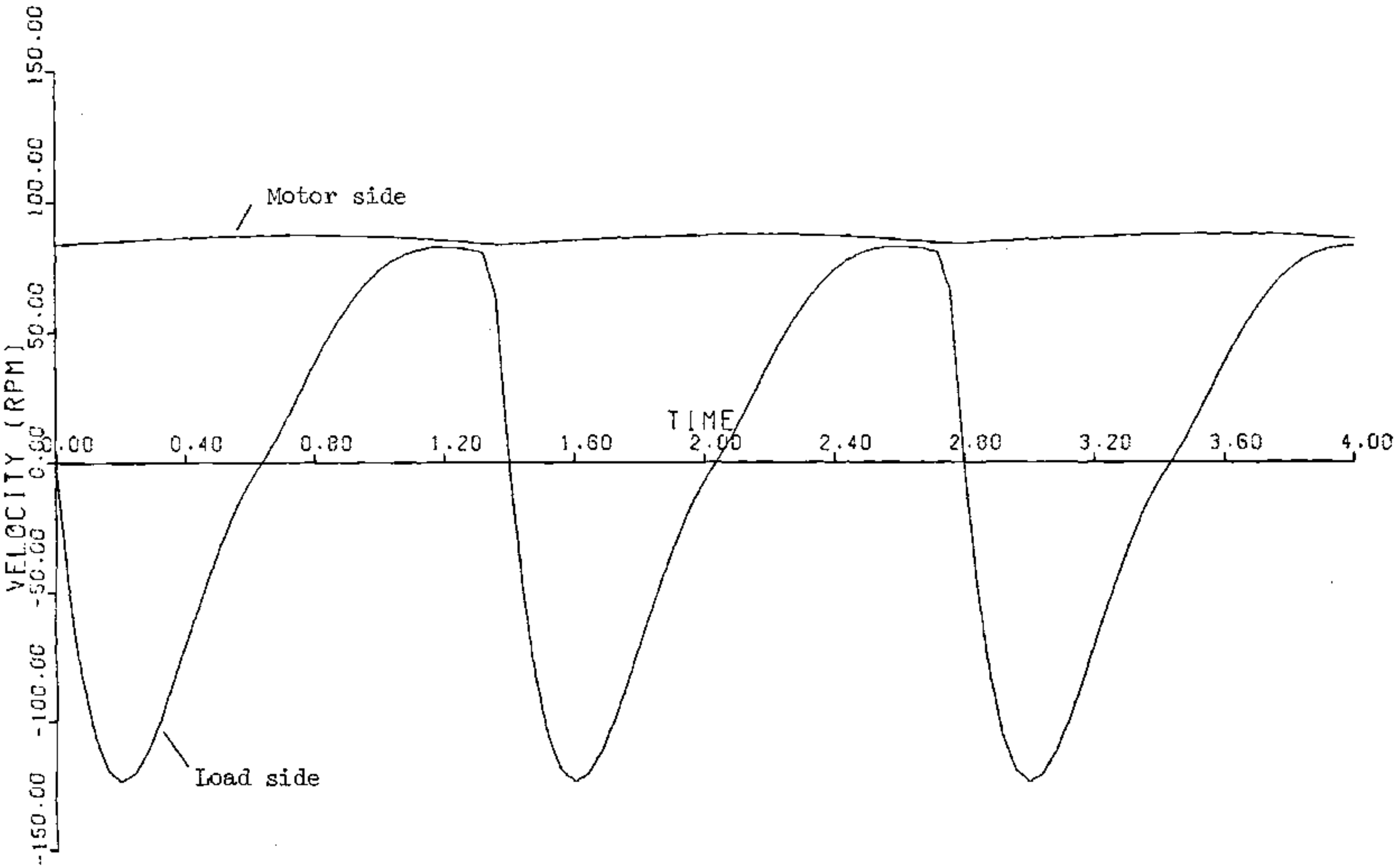


Figure 40. Motor Side and Load Side Clutch Velocities for FACTOR = 1.05.

Temperature Rise in the Clutch

Based on the development of Chapter IV and utilizing equations (17) and (18), estimates of the clutch maximum operating temperatures were calculated. It is worth repeating that those values are presented solely as an indication of whether heating is a problem or not, and do not represent a high degree of numerical accuracy.

For the system discussed earlier in this chapter, (FACTOR = 1.05),

$T_{\infty}'' = 155.2\ ^{\circ}F$ [Fazekaz]

$t_{av} = 138.8\ ^{\circ}F$ [Gagne]

$h = .65\ \text{Btu/sec-ft}^2\text{-}^{\circ}F$

The values obtained by the two equations agree well since the average value should be somewhat less than the ultimate temperature, and forecast that heating will not be a problem. Popular friction materials typically have temperature ratings of approximately 300°F, and manufacturers recommend a 100° margin below this value. Thus, even at 200°F, the rating is comfortably above the predicted ultimate tempaterure.

The correlation between relative slip and the temperature rise is illustrated by the following figures for the same system but with FACTOR = 1.1,

$T_{\infty}'' = 175.0\ ^{\circ}F$

$t_{av} = 154.7\ ^{\circ}F$

$h = .65\ \text{Btu/sec-ft}^2 - {}^{\circ}F$

Clutch heating presents a problem mainly because of its intimate connection with wear. Thermal degradation of the friction surfaces (caused by operating at or above the rated temperature) often is the primary contributor to short clutch life. However, as the above calculations indicate, normal wear may be expected to apply.

Wear in the Clutch

Unfortunately, calculating the expected wear in a clutch is very difficult. The current modeling of the wear process is rather inexact but even then, the resultant equations are extremely complex. Thus, the analytical methods leave something to be desired in the way of accuracy.

The most practical, and the most common, procedure for determining the wear rate of clutches is an application test, wherein a particular clutch (or friction material) is cycled a number of times under conditions consistant with the proposed application. The wear may then be directly measured with satisfactory accuracy.

Motor Heating

Most motor failures result from thermal breakdown of the winding insulation. This generally has catastrophic results with complete burn-out of the motor. Since a rewinding operation is costly, the end result is generally the purchase of a new motor.

As in the case for clutch heating, an allowable temperature rise is generally specified by the manufacturer. Fortunately, however, this requirement is automatically satisfied if the motor is operated at, or below, its full load rated horsepower in standard ambient conditions. Therefore, a temperature rise calculation for the motor is unnecessary as long as the load rating is met.

System Stability

For the system considered, the stability program was run for damping ratios of 89 and 78 percent of the design value. The results obtained from this program are most clearly presented in tabular form. The parameters of interest are the new maximum and minimum values of load angular displacement, the new arc of motion, the new period, the new upper and lower motor speeds, and the new horsepower loading associated with the change in damping. This material is presented in Table 1.

In general, the results seem intuitively sound - for reduced damping the period of oscillation is shorter and the total swing is larger. The drive side response is not predictable in general, however - although the damping is less, the motor must expend energy in opposing the load tendency to overshoot at θ_1. The resultant power loading stems from the complex interrelationship between alterations in the length of the engagement interval, the period, and even the manner in which the load velocity varies over the engagement interval.

The table does not indicate the number of cycles required to reach a steady operating mode after being released from the design initial conditions. For each test case, the motion was steady within one cycle. This is illustrated in Figure 41.

Similar promising results were obtained for the test wherein the motion was started from rest at the static equilibrium point. The motor accelerates the load until θ_2 is reached and then disengages, and the motion carries on in a similar fashion to the design case.

Table 1. Stability Results.

	Max θ (deg)	Min θ (deg)	Arc (deg)	Period (sec)	ω_1 (rpm)	ω_2 (rpm)	Hp
ζ_{Design}	239.5	-30.5	270	1.4	1747	1666	.41
$.89\zeta_D$	242	-42	284	1.22	1737	1669	.37
$.78\zeta_D$	245	-53	298	1.15	1736	1673	.34

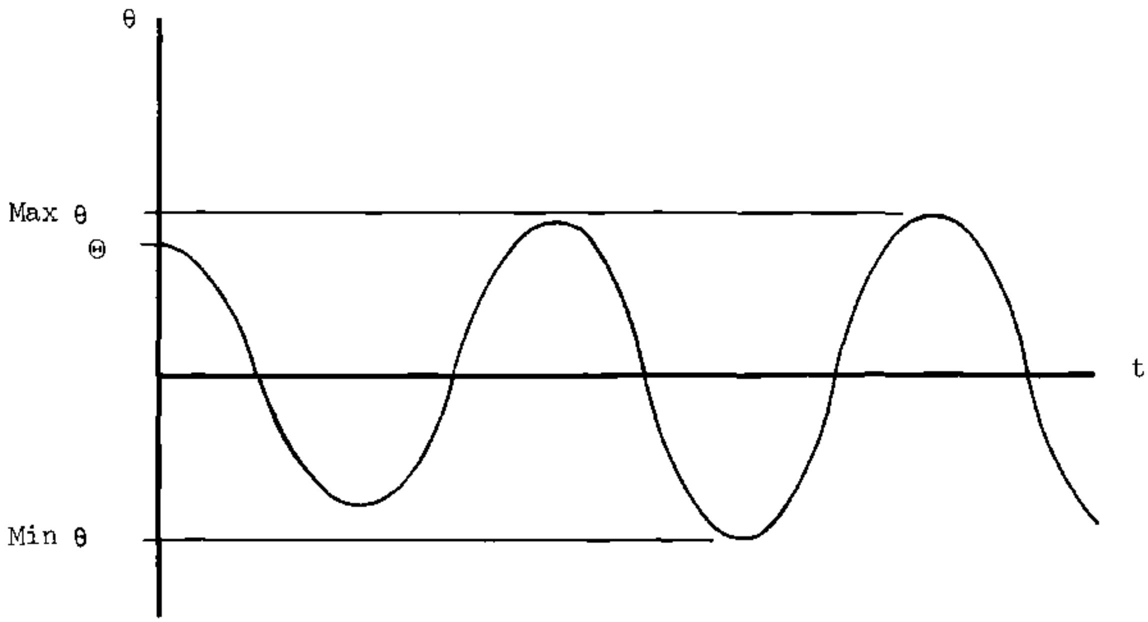


Figure 41. Motion for Damping Other Than Design Value.

Again, steady motion was obtained within one cycle.

On the basis of these results, signaling clutch actuation by load angular position seems mathematically appealing. This offers the simplest construction mechanically, making this method appear to be the best approach for a physical model.

In summary, the complete set of input and output data for a successful combination of parameters is shown in the form of computer print-out in Figure 42. Most of the terms have been described previously, however, a few need additional clarification.

In the input group, the allowable horsepower load is a figure set by the designer. It is intended to reflect a desired power loading for the motor and may very well be rated horsepower. However, as discussed earlier, it may be satisfactory to apply the manufacturer's suggested service factor in which case the allowable loading would be that factor times the rated power. Alternatively, it may be desirable to specify a power loading of <u>less</u> than the rated horsepower, which may be achieved. This number is not processed in the computer, but it forms the criterion for the final loop A wherein the designer may vary k to adjust the motor power load. If the total time for the agitate mode in a washing machine is of short duration, it may be allowable for the designer to specify a smaller motor and allow it to operate at a slight overload just during this mode, for instance.

The motor breakdown speed is also not processed in the computer. This parameter is an important value associated with any motor, and is supplied so that the results may be checked against it to be assured

```
GIVEN-                                                           DATA-
        DESIRED ARC ................................................ARC =      4.7124 (RAD)
        DESIRED PERIOD .............................................TAU =  1.40 (SEC)
        MAXIMUM LOAD DAMPING COEFFICIENT (INCLUDES
           BEARINGS,ETC. ON LOAD SIDE OF CLUTCH) ...................C =    28.8000 (IN-LBF-SEC)
        MAXIMUM TOTAL LOAD INERTIA (INCLUDES SHAFT
           AND LOAD SIDE OF CLUTCH) ................................JSUBT =   4.4400 (IN-LBF-SEC-SEC)
        ALLOWABLE HORSEPOWER LOAD ON MOTOR .........................HP =  .500
        MOTOR PERFORMANCE CURVE - SLOPE ............................ALPHA =    -1.5000 (IN-LBF-SEC/RAD)
                                  INTERCEPT ........................TQ =   280.00 (IN-LBF)
        MOTOR BREAKDOWN SPEED ......................................BRKDWN =   168.00 (RAD/SEC)
        FLYWHEEL INERTIA (CONSIDERS GEAR,PINION,
           AND MOTOR SIDE OF CLUTCH MASSLESS) ......................J =   1.0000 (IN-LBF-SEC-SEC)
        CLUTCH SLIP FACTOR .........................................FACTOR =  1.05


FINDS-                                                           RESULTS-
        TORSIONAL SPRING CONSTANT ......................................K =   155.00 (IN-LBF/RAD)
        STEP DOWN GEAR RATIO ...........................................R = 19.88
        RAMP TORQUE SLOPE ..............................................TR =  1179.69 (IN-LBF/SEC)
        TIME TO ENGAGE CLUTCH ..........................................T1 =     .6361 (SEC)
        TIME TO DISENGAGE CLUTCH .......................................T2 =    1.3515 (SEC)
        POSITION TO ENGAGE CLUTCH ......................................THETA1 =     -.53 (RAD)
        POSITION TO DISENGAGE CLUTCH ...................................THETA2 =     3.99 (RAD)
        LOAD ANGULAR VELOCITY AT DISENGAGEMENT .........................THET2D =     8.35 (RAD/SEC)
        MOTOR SPEED FLUCTUATION- HIGH ..................................OMEGA1 =   182.20 (RAD/SEC)
                                 LOW ...................................OMEGA2 =   174.20 (RAD/SEC)
        DAMPING RATIO ..................................................ZETA =  .55
        PEAK TORQUE TRANSMITTED ........................................TQMAX =   843.75 (IN-LBF)
        TIME INTERVAL FOR TRANSMISSION .................................TRANS =    .7152 (SEC)
        TIME INTERVAL FOR ACCELERATING .................................ACCEL =    .8848 (SEC)
        WORK PER CYCLE DONE BY MOTOR ...................................WORK =  3768.12 (IN-LBF)
        AVERAGE HORSEPOWER .............................................HPWR =  .4078
        INITIAL ANGULAR DISPLACEMENT ...................................CAPTHE =     4.18 (RAD)
        HEAT GENERATED PER CYCLE .......................................Q =  7.0679+02 (IN-LBF)
```

Figure 42. Computer Output for Successful Set of Parameters.

of operating in a safe portion of the running curve.

In the output group, the peak torque transmitted is the maximum value that the ramp torque Trt builds up to during engagement. The remaining results are self explanatory.

This particular computer output is the best combination of parameters that was achieved for a particular motor. Emphasis is placed on the fact that it is not an optimum design, however. The many other data sets obtained throughout the period of research helped arrive at the final program form. The great bulk of the data proved useful in determining trends established by varying certain parameters. Portions of this early output and analysis deemed helpful are provided in Appendix C for reference.

The use of energy storage concepts in reducing the motor loading necessary to oscillate a heavily damped, high inertia load has been demonstrated to be feasible. The large number of design variables precludes specifying the optimum design at this stage of research. However, as the primary tool for building a physical model, this work is intended to guide future researchers toward this goal.

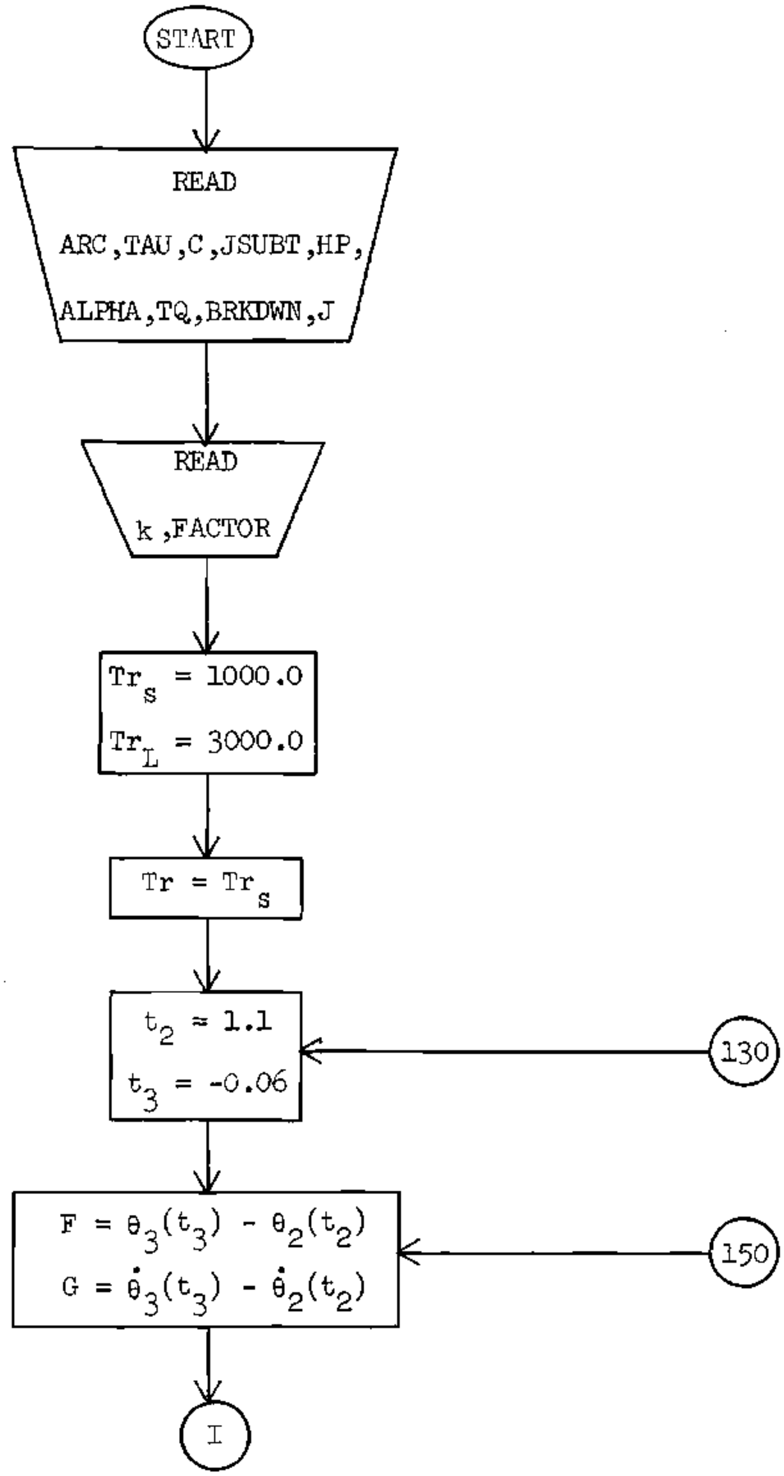
Recommendations

Future research in this area should be directed at optimizing the large number of design parameters. It would be profitable to investigate the use of a smaller motor and/or a larger flywheel, for example. The area of clutch modeling requires further study to correlate the solutions predicted mathematically here with data obtained from a prototype design. The clutch may be located on either side of the reduction with equal mathematical results so both arrangements should be considered in the physical design. The physical layout of the clutch, reduction unit, torsional spring, and the clutch actuator mechanism must be chosen to meet performance specifications as well as practical construction and cost techniques. Therefore, the work presented here should be considered as the basic design tool in guiding future research in constructing a prototype machine.

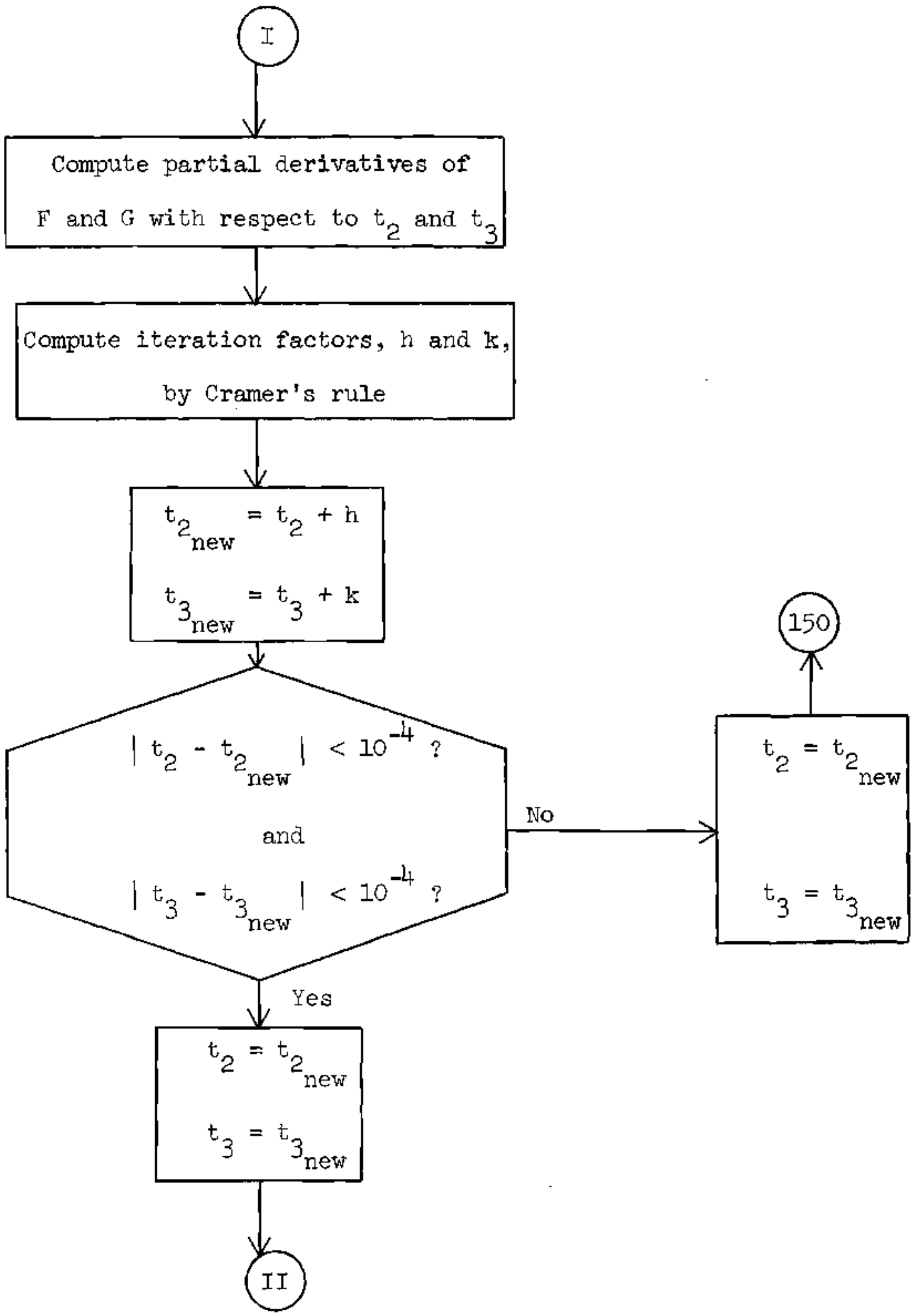
APPENDIX A

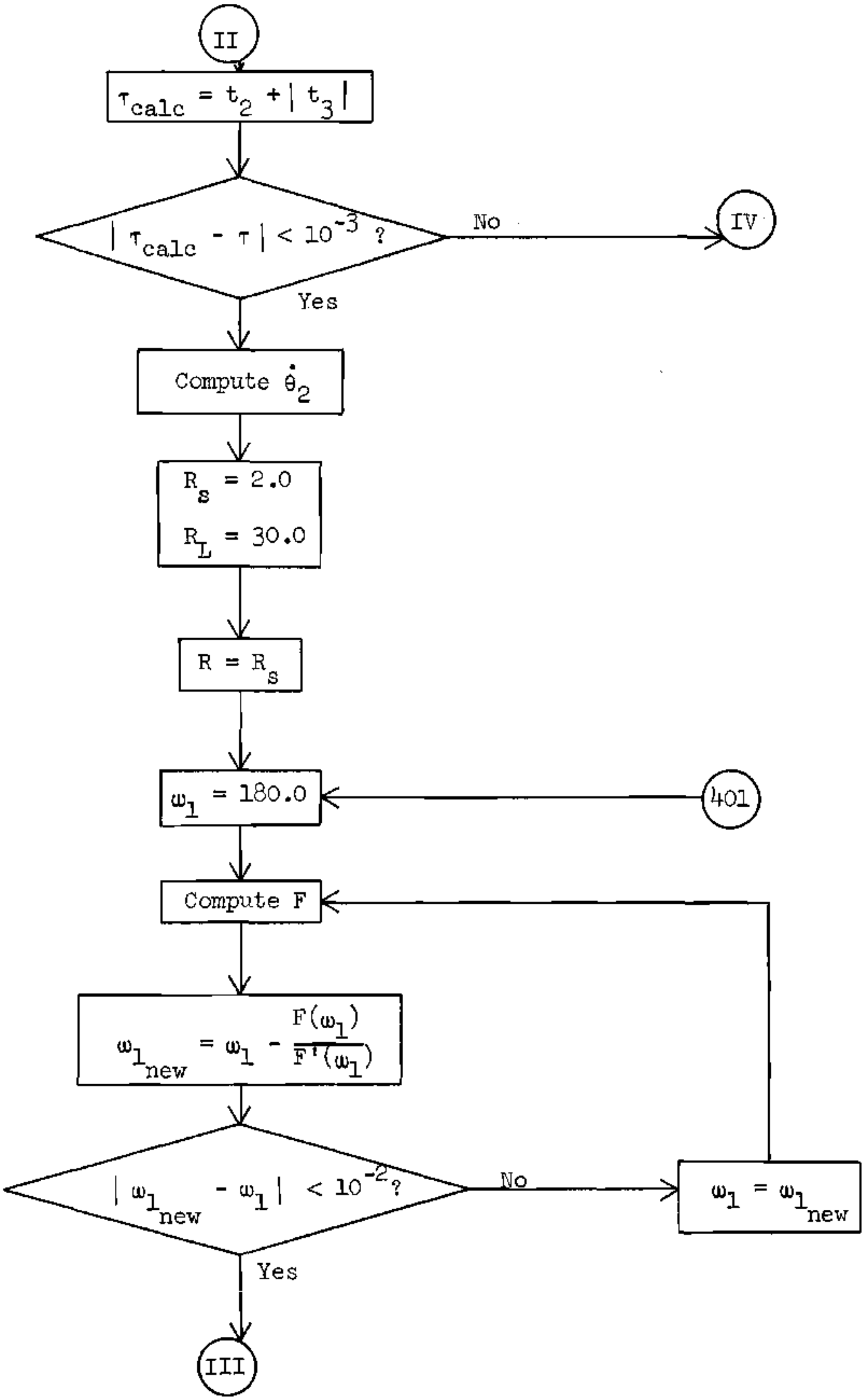
MAIN PROGRAM

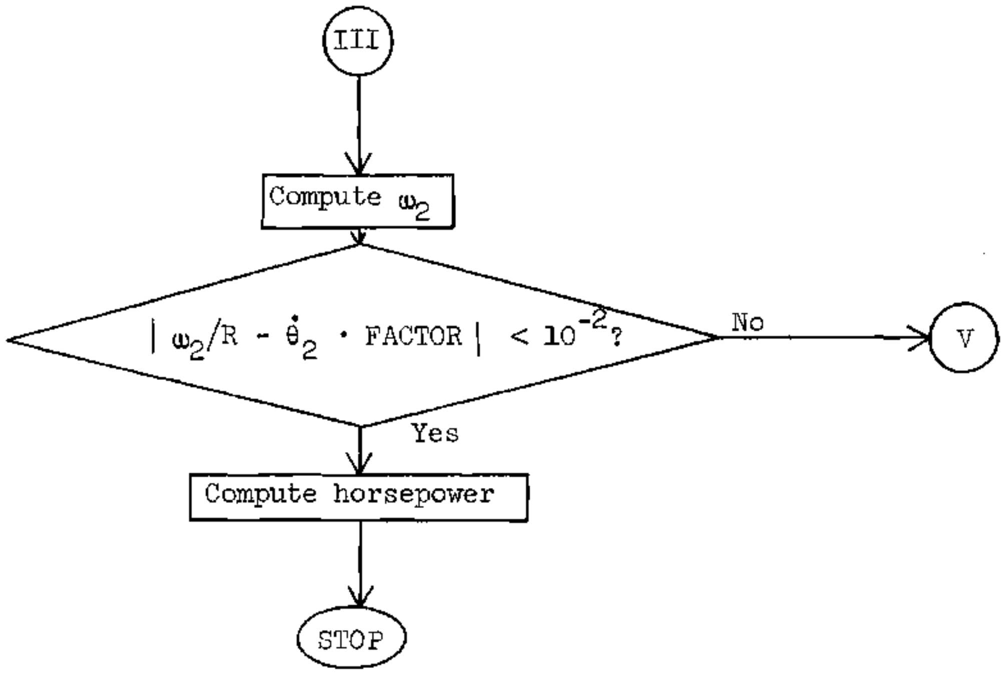
A flow chart for the main program as well as the actual program in FORTRAN IV language are reproduced on the following pages.

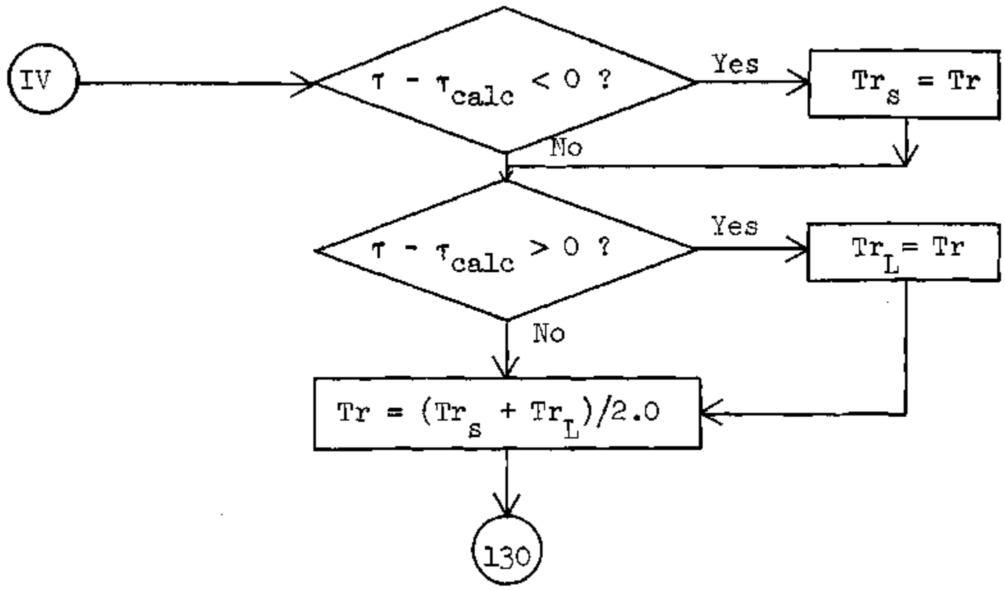


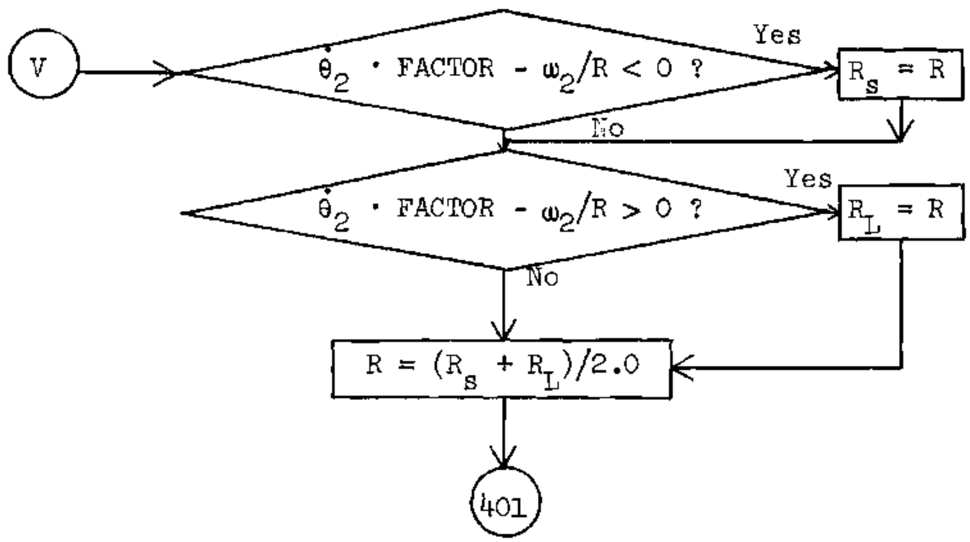
START
READ
ARC,TAU,C,JSUBT,HP,
ALPHA,TQ,BRKDWN,J
READ
k,FACTOR
$Tr_s = 1000.0$
$Tr_L = 3000.0$
$Tr = Tr_s$
$t_2 = 1.1$
$t_3 = -0.06$
130
$F = \theta_3(t_3) - \theta_2(t_2)$
$G = \dot{\theta}_3(t_3) - \dot{\theta}_2(t_2)$
150
I


I
Compute partial derivatives of F and G with respect to t_2 and t_3
Compute iteration factors, h and k, by Cramer's rule
$t_{2_{new}} = t_2 + h$
$t_{3_{new}} = t_3 + k$
$|t_2 - t_{2_{new}}| < 10^{-4}$?
and
$|t_3 - t_{3_{new}}| < 10^{-4}$?
No
$t_2 = t_{2_{new}}$
$t_3 = t_{3_{new}}$
150
Yes
$t_2 = t_{2_{new}}$
$t_3 = t_{3_{new}}$
II


II
$\tau_{calc} = t_2 + |t_3|$
$|\tau_{calc} - \tau| < 10^{-3}$?
No
IV
Yes
Compute $\dot{\theta}_2$
$R_s = 2.0$
$R_L = 30.0$
$R = R_s$
$\omega_1 = 180.0$
401
Compute F
$\omega_{1_{new}} = \omega_1 - \frac{F(\omega_1)}{F'(\omega_1)}$
$|\omega_{1_{new}} - \omega_1| < 10^{-2}$?
No
$\omega_1 = \omega_{1_{new}}$
Yes
III


III
Compute ω_2
$|\omega_2/R - \dot{\theta}_2 \cdot \mathrm{FACTOR}| < 10^{-2}$?
No
V
Yes
Compute horsepower
STOP


IV
$\tau - \tau_{calc} < 0$?
Yes
$Tr_s = Tr$
No
$\tau - \tau_{calc} > 0$?
Yes
$Tr_L = Tr$
No
$Tr = (Tr_s + Tr_L)/2.0$
130


V
$\dot{\theta}_2 \cdot \text{FACTOR} - \omega_2/R < 0$?
Yes
$R_s = R$
No
$\dot{\theta}_2 \cdot \text{FACTOR} - \omega_2/R > 0$?
Yes
$R_L = R$
No
$R = (R_s + R_L)/2.0$
401

```
-RUN FORTN,01E25602,HORN-W-A,1,100
-ASG,T PLOT,T
-USE 3,PLOT
-MSG PLOT CHARGES TO 01E25602
-FOR,IS MAIN
C  RAMP INPUT
      IMPLICIT REAL (A-Z)
      INTEGER N
      INTEGER I,IBUFF,M,U
      INTEGER I1,I2,I1M,I2P
      DIMENSION IBUFF(1400),THETA(101),TIME(101),THETAD(101)
      DIMENSION OMEGR(101),TUBVEL(101),INTGND(101)
      READ (5,100,END=200) ARC,TAU,C,JSUBT,HP,ALPHA,TQ,BRKDWN,J
  100 FORMAT (9F8.0)
C
  148 READ (5,149) K,FACTOR
  149 FORMAT (2F10.0)
      IF (K .LT. 0.0) GO  TO  200
C
C     LOOP B BEGINS. FINDS TR.
C
  402 ZETA = C / (2.0 * SQRT(JSUBT * K))
      OMEGAN =SQRT(K / JSUBT)
      OMEGAD = OMEGAN * SQRT(1.0 - ZETA**2)
      T1 = 3.14159 / OMEGAD
      CAPTHE = ARC / (1.0 + EXP(-ZETA * OMEGAN * T1))
      DEG = CAPTHE * 57.2958
      ZETAWN = ZETA * OMEGAN
C
C     PICK UPPER AND LOWER EXTREMUMS ON TR.
C
      TRS = 1000.0
      TRL = 3000.0
      TR = TRS
C
C     LOOP C BEGINS. FINDS T2 AND T3.
C     PICK INITIAL T2 AND T3.
C
  130 T2 = 1.1
      T3 = -0.06
C
C     COMPUTE ARGUMENTS, ETC.
  150 ARG1 = OMEGAD * T2
      ARG2 = OMEGAD * (T2 - T1)
      ARG3 = OMEGAD * T3
      RADICL  = SQRT(1.0 - ZETA**2)
      THET2  = CAPTHE * EXP(-ZETAWN * T2) * (COS(ARG1) + ZETA / RADICL *
     A SIN(ARG1)) + TR / K * ((T2 - T1) - 2.0 * ZETA / OMEGAN * (1.0 - E
     B XP(-ZETAWN * (T2 - T1)) * (COS(ARG2) + (ZETA**2 - 0.5) /(ZETA * R
     C ADICL) * SIN(ARG2))))
C
      THET2D = TR / K * (1.0 - ZETA * EXP(-ZETAWN * (T2 - T1)) * (1.0 /
     A RADICL * SIN(ARG2) + 1.0 / ZETA * COS(ARG2))) - CAPTHE * OMEGAN /
     B RADICL * EXP(-ZETAWN * T2) * SIN(ARG1)
C
      THET3 = CAPTHE * EXP(-ZETAWN * T3) * (COS(ARG3) + ZETA / RADICL *
     A SIN(ARG3))
C
      THET3D = - CAPTHE * EXP(-ZETAWN * T3) * (OMEGAN / RADICL * SIN(ARG
     A 3))
```

```
C
      F = THET3 - THET2
      G = THET3D - THET2D
C
C  FSUBT2 IS PARTIAL OF F WRT T2
      FSUBT2 = - THET2D
C
C  FSUBT3 IS PARTIAL OF F WRT T3
C
      FSUBT3 = THET3D
C
C  GSUBT2 IS PARTIAL OF G WRT T2
C
      GSUBT2 = CAPTHE * OMEGAN**2 * EXP(-ZETAWN * T2) * (COS(ARG1) - ZET
     A A / RADICL * SIN(ARG1)) - (TR * OMEGAN) / (K * RADICL) * EXP(-ZET
     B AWN * (T2 - T1)) * SIN(ARG2)
C
C  GSUBT3 IS PARTIAL OF G WRT T3
C
      GSUBT3 = CAPTHE * OMEGAN / RADICL * EXP(-ZETAWN * T3) * (ZETAWN *
     A SIN(ARG3) - OMEGAD * COS(ARG3))
C
C  SOLVE FOR H AND KAY BY CRAMERS RULE
      DENOM = FSUBT2 * GSUBT3 - GSUBT2 * FSUBT3
      H = (-F * GSUBT3 + G * FSUBT3) / DENOM
      KAY = (-G * FSUBT2 + F * GSUBT2) / DENOM
C  CHECK FOR ZERO OR NEAR ZERO DENOM
      IF (ABS(H).GE. 1.E6 .OR. ABS(KAY) .GE. 1.E6) GO TO 199
C  END OF CRAMERS RULE
      T2NEW = T2 + H
      T3NEW = T3 + KAY
      IF (ABS(T2 - T2NEW) .LT. 1.E-4 .AND. ABS(T3 - T3NEW) .LT. 1.E-4) G
     C O TO 180
      T2 = T2NEW
      T3 = T3NEW
      GO TO 150
  180 T2 = T2NEW
      T3 = T3NEW
      PERIOD = T2 + ABS(T3)
C
C     END OF LOOP C.
      IF (ABS(PERIOD - TAU) .LT. 1.E-3) GO TO 198
      IF (TR .EQ. TRS) GO TO 13
      IF (TR .EQ. TRL) PERDS = PERIOD
      IF ((TAU - PERDS) .LT. 0.0) GO TO 11
      IF ((TAU - PERDL) .GT. 0.0) GO TO 15
      IF (TAU - PERIOD .LT. 0.0) TRS = TR
      IF (TAU - PERIOD .GT. 0.0) TRL = TR
      TR = (TRS + TRL) / 2.0
      GO TO 130
  13  PERDL = PERIOD
      TR = TRL
      GO TO 130
  11  WRITE (6,14)
   14 FORMAT (1X,-TRL TOO SMALL-)
      GO TO 200
  15  WRITE (6,16)
   16 FORMAT (1X,-TRS TOO LARGE-)
      GO TO 200
  199 WRITE (6,178)
```

```
  178 FORMAT (1X,-ZERO DENOM-)
      GO TO 200
  198 ARG1 = OMEGAD * T2
      ARG2 = OMEGAD * (T2 - T1)
      RADICL = SQRT(1.0 - ZETA**2)
      THET2  = CAPTHE * EXP(-ZETAWN * T2) * (COS(ARG1) + ZETA / RADICL *
     A SIN(ARG1)) + TR / K * ((T2 - T1) - 2.0 * ZETA / OMEGAN * (1.0 - E
     B XP(-ZETAWN * (T2 - T1)) * (COS(ARG2) + (ZETA**2 - 0.5) /(ZETA * R
     C ADICL) * SIN(ARG2))))
      THET2D = TR / K * (1.0 - ZETA * EXP(- ZETAWN * (T2 - T1)) * (1.0 /
     A RADICL * SIN(ARG2) + 1.0 /ZETA * COS(ARG2))) - CAPTHE * OMEGAN /
     B RADICL * EXP(-ZETAWN * T2) * SIN(ARG1)
C
C      END OF LOOP B.
C      LOOP D BEGINS. FINDS R.
C
C      PICK UPPER AND LOWER EXTREMUMS ON R.
      RS = 2.0
      RL = 30.0
C
      R = RS
C
C      LOOP E BEGINS. FINDS OMEG1.
C
C      PICK INITIAL OMEG1.
C
  401  OMEG1 = 180.0
C
      TRANS = T2 - T1
      ACCEL = TAU - TRANS
      N = 1
  107 OMEG2 = (OMEG1 + TQ / ALPHA - (TR * J) / (R * ALPHA**2)) * EXP(ALP
     A HA / J * TRANS) + TR / (R * ALPHA) * TRANS + (TR * J / (R * ALPHA
     B **2) - TQ / ALPHA)
      F = (-TQ / ALPHA + (TQ / ALPHA + OMEG2) * EXP(ALPHA / J * ACCEL))-
     A OMEG1
      DF = EXP(ALPHA / J * (ACCEL + TRANS)) - 1.0
      IF (F / DF .GT. 1.E6) GO TO 170
      OMEG1N = OMEG1 - (F / DF)
      IF (ABS(OMEG1N - OMEG1) .LT. 1.E-2) GO TO 120
      N = N + 1
      IF (N .GT. 15) GO TO 140
      OMEG1 = OMEG1N
      GO TO 107
  140 WRITE (6,138)
  138 FORMAT (1X,-N TOO BIG-)
      GO TO 200
  120 OMEG1 = OMEG1N
      OMEG2 = (OMEG1 + TQ / ALPHA - (TR * J) / (R * ALPHA**2)) * EXP(ALP
     A HA / J * TRANS) + TR / (R * ALPHA) * TRANS + (TR * J / (R * ALPHA
     B **2) - TQ / ALPHA)
C      END OF LOOP E.
C
      IF (ABS(OMEG2/R - THET2D * FACTOR) .LT. 1.E-2) GO TO 408
      IF (R .EQ. RS) GO TO 78
      IF (R .EQ. RL) VELS = OMEG2 / R
      IF ((THET2D * FACTOR - VELS) .LT. 0.0) GO TO 79
      IF ((THET2D * FACTOR - VELL) .GT. 0.0) GO TO 80
      IF (THET2D * FACTOR - OMEG2/R .LT. 0.0) RS = R
      IF (THET2D * FACTOR - OMEG2/R .GT. 0.0) RL = R
```

```
      R = (RS + RL) / 2.0
      GO TO 401
   78 VELL = OMEG2 / R
      R = RL
      GO TO 401
   79 WRITE (6,81)
   81 FORMAT (1X,-RL TOO SMALL-)
      GO TO 200
   80 WRITE (6,82)
   82 FORMAT (1X,-RS TOO LARGE-)
      GO TO 200
  408 T3 = TAU + T1
      A1 = - (TQ / ALPHA + OMEG2)
      B1 = - TQ / ALPHA
      C1 = OMEG1 + TQ / ALPHA - (TR * J) / (R * ALPHA**2)
      D1 = TR / (R * ALPHA)
      E1 = (TR * J) / (R * ALPHA**2) - TQ / ALPHA
      F1 = (TR * J) / (R * ALPHA**2)
      WORK = - ALPHA * (C1**2 * (J / (2.0 * -ALPHA)) * (EXP((2.0 * ALPHA
     A ) / J * T2) - EXP((2.0 * ALPHA) / J * T1)) + 2.0 * C1 * D1 * (J /
     B ALPHA)**2 * ((1.0 - ALPHA / J * T2) * EXP(ALPHA / J * T2) - (1.0
     C - ALPHA / J * T1) * EXP(ALPHA / J * T1)) + C1 * (E1 + F1) * (J /
     D - ALPHA) * (EXP(ALPHA / J * T2) - EXP(ALPHA / J * T1)) + 1.0 / 3.
     E * D1**2 * (T1**3 - T2**3) + 0.5 * D1 * (E1 + F1) * (T1**2 - T2**2
     F ) + E1 * F1 * (T1 - T2)) + (TQ * B1 + ALPHA * B1**2) * (T3 - T2)
     G - (TQ * A1 * J / ALPHA + 2.0 * B1 * A1 * J) * (EXP(ALPHA / J * T3
     H ) - EXP(ALPHA / J * T2)) + A1**2 * J / 2.0 * (EXP(2.0 * ALPHA / J
     I * T3) - EXP(2.0 * ALPHA / J * T2))
C
C     COMPUTE AVERAGE HORSEPOWER
      HPWR = WORK * (1.0 / 12.0) * (1.0 / TAU) * (1.0 / 550.0)
C     END LOOP D.
      GO TO 409
  170 WRITE (6,171)
  171 FORMAT (1X,-ZERO DENOM 2-)
      GO TO 200
  409 WRITE (6,500)
  500 FORMAT (1H1,1X,-GIVEN--,62X,-DATA--)
      WRITE (6,501) ARC
  501 FORMAT (7X,-DESIRED ARC -,55(1H.),-ARC = -,F10.4,1X,-(RAD)-)
      WRITE (6,502) TAU
  502 FORMAT (7X,-DESIRED PERIOD -,52(1H.),-TAU = -,F5.2,1X,-(SEC)-)
      WRITE (6,503)
  503 FORMAT (7X,-MAXIMUM LOAD DAMPING COEFFICIENT (INCLUDES-)
      WRITE (6,504) C
  504 FORMAT (11X,-BEARINGS,ETC. ON LOAD SIDE OF CLUTCH) -,25(1H.),-C =
     C-,F10.4,1X,-(IN-LBF-SEC)-)
      WRITE (6,505)
  505 FORMAT (7X,-MAXIMUM TOTAL LOAD INERTIA (INCLUDES SHAFT-)
      WRITE (6,506) JSUBT
  506 FORMAT (11X,-AND LOAD SIDE OF CLUTCH) -,38(1H.),-JSUBT = -,F8.4,1X
     C,-(IN-LBF-SEC-SEC)-)
      WRITE (6,507) HP
  507 FORMAT (7X,-ALLOWABLE HORSEPOWER LOAD ON MOTOR -,32(1H.),-HP = -,F
     C5.3)
      WRITE (6,508) ALPHA
  508 FORMAT (7X,-MOTOR PERFORMANCE CURVE - SLOPE -,35(1H.),-ALPHA = -,F
     C10.4,1X,-(IN-LBF-SEC/RAD)-)
      WRITE (6,509) TQ
  509 FORMAT (33X,-INTERCEPT -,31(1H.),-TQ = -,F8.2,1X,-(IN-LBF)-)
```

```
      WRITE (6,510) BRKDWN
  510 FORMAT (7X,-MOTOR BREAKDOWN SPEED -,45(1H.),-BRKDWN = -,F8.2,1X,-(
     CRAD/SEC)-)
      WRITE (6,511)
  511 FORMAT (7X,-FLYWHEEL INERTIA (CONSIDERS GEAR,PINION,-)
      WRITE (6,512) J
  512 FORMAT (11X,-AND MOTOR SIDE OF CLUTCH MASSLESS) -,28(1H.),-J = -,F
     C8.4,1X,-(IN-LBF-SEC-SEC)-)
      WRITE (6,532) FACTOR
  532 FORMAT (7X,-CLUTCH SLIP FACTOR -,48(1H.),-FACTOR = -,F5.2)
      WRITE (6,513)
  513 FORMAT (/////,1X,-FINDS--,62X,-RESULTS--)
      WRITE (6,514) K
  514 FORMAT (7X,-TORSIONAL SPRING CONSTANT -,44(1H.),-K = -,F8.2,1X,-(I
     CN-LBF/RAD)-)
      WRITE (6,515) R
  515 FORMAT (7X,-STEP DOWN GEAR RATIO -,49(1H.),-R = -,F5.2)
      WRITE (6,516) TR
  516 FORMAT (7X,-RAMP TORQUE SLOPE -,52(1H.),-TR = -,F8.2,1X,-(IN-LBF/S
     CEC)-)
      WRITE (6,517) T1
  517 FORMAT (7X,-TIME TO ENGAGE CLUTCH -,48(1H.),-T1 = -,F8.4,1X,-(SEC)
     C-)
      WRITE (6,518) T2
  518 FORMAT (7X,-TIME TO DISENGAGE CLUTCH -,45(1H.),-T2 = -,F8.4,1X,-(S
     CEC)-)
      THETA1 = CAPTHE * EXP(-ZETAWN * T1) * (COS(OMEGAD * T1) + ZETA / R
     C ADICL * SIN(OMEGAD * T1))
      WRITE (6,519) THETA1
  519 FORMAT (7X,-POSITION TO ENGAGE CLUTCH -,44(1H.),-THETA1 = -,F8.2,1
     CX,-(RAD)-)
      WRITE (6,520) THET2
  520 FORMAT (7X,-POSITION TO DISENGAGE CLUTCH -,41(1H.),-THETA2 = -,F8.
     C2,1X,-(RAD)-)
      WRITE (6,521) THET2D
  521 FORMAT (7X,-LOAD ANGULAR VELOCITY AT DISENGAGEMENT -,31(1H.),-THET
     C2D = -,F8.2,1X,-(RAD/SEC)-)
      WRITE (6,522) OMEG1
  522 FORMAT (7X,-MOTOR SPEED FLUCTUATION- HIGH -,40(1H.),-OMEGA1 = -,F8
     C.2,1X,-(RAD/SEC)-)
      WRITE (6,523) OMEG2
  523 FORMAT (32X,-LOW -,41(1H.),-OMEGA2 = -,F8.2,1X,-(RAD/SEC)-)
      WRITE (6,524) ZETA
  524 FORMAT (7X,-DAMPING RATIO -,56(1H.),-ZETA = -,F4.2)
      TQMAX = TR * TRANS
      WRITE (6,525) TQMAX
  525 FORMAT (7X,-PEAK TORQUE TRANSMITTED -,46(1H.),-TQMAX = -,F8.2,1X,-
     C(IN-LBF)-)
      WRITE (6,526) TRANS
  526 FORMAT (7X,-TIME INTERVAL FOR TRANSMISSION -,39(1H.),-TRANS = -,F8
     C.4,1X,-(SEC)-)
      WRITE (6,527) ACCEL
  527 FORMAT (7X,-TIME INTERVAL FOR ACCELERATING -,39(1H.),-ACCEL = -,F8
     C.4,1X,-(SEC)-)
      WRITE (6,528) WORK
  528 FORMAT (7X,-WORK PER CYCLE DONE BY MOTOR -,41(1H.),-WORK = -,F8.2,
     C1X,-(IN-LBF)-)
      WRITE (6,529) HPWR
  529 FORMAT (7X,-AVERAGE HORSEPOWER -,51(1H.),-HPWR = -,F6.4)
      WRITE (6,530) CAPTHE
```

```
  530 FORMAT (7X,-INITIAL ANGULAR DISPLACEMENT -,41(1H.),-CAPTHE = -,F8.
     C2,1X,-(RAD)-)
C   BEGIN PLOT SEGMENT FOR DISPLACEMENT.
      DO 1 I = 1,101
      TIME(I) = I - 1
      TIME(I) = TIME(I) * 0.04
      IF (TIME(I) .GT. T1) GO TO 2
      M = 1
  5   ARG4 = OMEGAD * TIME(I)
      ARG5 = OMEGAD * (TIME(I) - T1)
      THETA(I) = (CAPTHE * EXP(-ZETAWN * TIME(I)) * (COS(ARG4) + ZETA /
     A RADICL * SIN(ARG4)))
      THETA(I) = (THETA(I) * 57.2958) * 0.01
      GO TO (1,801,802,803),M
   2  IF (TIME(I) .GT. T2) GO TO 3
      M = 1
  7   ARG4 = OMEGAD * TIME(I)
      ARG5 = OMEGAD * (TIME(I) - T1)
      THETA(I) = (CAPTHE * EXP(-ZETAWN * TIME(I)) * (COS(ARG4) + ZETA /
     A RADICL * SIN(ARG4)) + TR / K * ((TIME(I) - T1) - 2.0 * ZETA / OME
     BGAN * (1.0 - EXP(-ZETAWN * (TIME(I) - T1)) * (COS(ARG5) + (ZETA**2
     C - 0.5) / (ZETA * RADICL) * SIN(ARG5)))))
      THETA(I) = (THETA(I) * 57.2958) * 0.01
      GO TO (1,801,802,200),M
   3  IF (TIME(I) .GT. PERIOD + T1) GO TO 4
      M = 2
      TIME(I) = TIME(I) - PERIOD
      GO TO 5
   4  IF (TIME(I) .GT. PERIOD + T2) GO TO 6
      M = 2
      TIME(I) = TIME(I) - PERIOD
      GO TO 7
    6 IF (TIME(I) .GT. (2.0 * PERIOD + T1)) GO TO 600
      TIME(I) = TIME(I) - 2.0 * PERIOD
      M = 3
      GO TO 5
  600 IF (TIME(I) .GT. (2.0 * PERIOD + T2)) GO TO 601
      TIME(I) = TIME(I) - 2.0 * PERIOD
      M = 3
      GO TO 7
  601 TIME(I) = TIME(I) - 3.0 * PERIOD
      M = 4
      GO TO 5
  801 TIME(I) = TIME(I) + PERIOD
      GO TO 1
  802 TIME(I) = TIME(I) + 2.0 * PERIOD
      GO TO 1
  803 TIME(I) = TIME(I) + 3.0 * PERIOD
    1 TIME(I) = TIME(I) * 2.5
      CALL PLOTS(IBUFF,1400,3)
      CALL PLOT (1.0,5.5,-3)
      CALL AXIS (0.0,0.0,-TIME-,4,10.0,0.0,0.0,0.4)
      CALL AXIS (0.0,-3.0,-THETA (DEG)-,11,6.0,90.0,-300.0,100.0)
      CALL PLOT (TIME(1),THETA(1),3)
      DO 413 I = 2,101
  413 CALL PLOT(TIME(I),THETA(I),2)
C
C   BEGIN PLOT SEGMENT FOR VELOCITIES.
      CALL PLOTS(IBUFF,1400,3)
      CALL PLOT (12.0,0.0,-3)
```

```
      CALL AXIS (0.0,0.0,-TIME-,4,10.0,0.0,0.0,0.4)
      CALL AXIS (0.0,-3.0,-VELOCITY (RPM)-,14,6.0,90.0,-150.0,50.0)
      U = 0
 3115 DO 1111 I = 1,101
      TIME(I) = I - 1
      TIME(I) = TIME(I) * 0.04
      IF (TIME(I) .GT. T1) GO TO 2000
      M = 1
 5000 ARG4 = OMEGAD * TIME(I)
      ARG5 = OMEGAD * (TIME(I) - T1)
      IF (U .EQ. 1) GO TO 3111
      THETAD(I) = - CAPTHE * EXP(-ZETAWN * TIME(I)) * (OMEGAN / RADICL
     A * SIN(ARG4))
 3112 THETAD(I) = THETAD(I) * (60.0 / 6.28319) * 0.02
      GO TO (1000,8010,8020,8030),M
 2000 IF (TIME(I) .GT. T2) GO TO 3000
      M = 1
 7000 ARG4 = OMEGAD * TIME(I)
      ARG5 = OMEGAD * (TIME(I) - T1)
      IF (U .EQ. 1) GO TO 3114
      THETAD(I) = TR / K * (1.0 - ZETA * EXP(-ZETAWN * (TIME(I) - T1)) *
     A (1.0 / RADICL * SIN(ARG5) + 1.0 / ZETA * COS(ARG5))) - CAPTHE * O
     B MEGAN / RADICL * EXP(-ZETAWN * TIME(I)) * SIN(ARG4)
 3113 THETAD(I) = THETAD(I) * (60.0 / 6.28319) * 0.02
      GO TO (1000,8010,8020,200),M
 3000 IF (TIME(I) .GT. PERIOD + T1) GO TO 4000
      M = 2
      TIME(I) = TIME(I) - PERIOD
      GO TO 5000
 4000 IF (TIME(I) .GT. PERIOD + T2) GO TO 6000
      M = 2
      TIME(I) = TIME(I) - PERIOD
      GO TO 7000
 6000 IF (TIME(I) .GT. (2.0 * PERIOD + T1)) GO TO 6001
      TIME(I) = TIME(I) - 2.0 * PERIOD
      M = 3
      GO TO 5000
 6001 IF (TIME(I) .GT. (2.0 * PERIOD + T2)) GO TO 6010
      TIME(I) = TIME(I) - 2.0 * PERIOD
      M = 3
      GO TO 7000
 6010 TIME(I) = TIME(I) - 3.0 * PERIOD
      M = 4
      GO TO 5000
 8010 TIME(I) = TIME(I) + PERIOD
      GO TO 1000
 8020 TIME(I) = TIME(I) + 2.0 * PERIOD
      GO TO 1000
 8030 TIME(I) = TIME(I) + 3.0 * PERIOD
 1000 IF (U .EQ. 1) OMEGR(I) = THETAD(I)
      IF (U .EQ. 0) TUBVEL(I) = THETAD(I)
 1111 TIME(I) = TIME(I) * 2.5
      CALL PLOT (TIME(1),THETAD(1),3)
      DO 4130 I = 2,101
 4130 CALL PLOT (TIME(I),THETAD(I),2)
      IF (U .EQ. 1) GO TO 3116
      U = 1
      GO TO 3115
 3111 THETAD(I) = (-TQ / ALPHA + (TQ / ALPHA + OMEG2) * EXP(ALPHA / J *
     A (TIME(I) + (PERIOD - T2)))) / R
```

```
      GO TO 3112
 3114 THETAD(I) =((OMEG1 + TQ / ALPHA - (TR * J) / (R * ALPHA**2)) * EXP
     A (ALPHA / J * (TIME(I) - T1)) + TR / (R * ALPHA) * (TIME(I) - T1)
     B + (TR * J / (R * ALPHA**2) - TQ / ALPHA)) / R
      GO TO 3113
 3116 CALL PLOT (11.0,0.0,999)
C
C     THIS SEGMENT CALCULATES HEAT GENERATED IN CLUTCH.
      I1 = INT(T1 / 0.04) + 3
      I2 = INT(T2 / 0.04) + 1
      SUM = 0.0
      DO 691 I = I1,I2
      TIME(I) = TIME(I) / 2.5
      OMEGR(I) = (OMEGR(I) / 0.02) * (6.28319 / 60.)
      TUBVEL(I) = (TUBVEL(I) / 0.02) * (6.28319 / 60.)
      INTGND(I) = TR * (TIME(I) - T1) * (OMEGR(I) - TUBVEL(I))
  691 SUM = SUM + INTGND(I)
      I1M = I1 - 1
      I2P = I2 + 1
      INTGND(I1M) = TR * (TIME(I1M) - T1) * (OMEGR(I1M) - TUBVEL(I1M)) *
     C(6.28319 / 60.) * (1.0 / 0.02)
      INTGND(I2P) = TR * (TIME(I2P) - T1) * (OMEGR(I2P) - TJBVEL(I2P)) *
     C(6.28319 / 60.) * (1.0 / 0.02)
      Q = 0.02 * (INTGND(I1M) + 2.0 * SUM + INTGND(I2P))
      WRITE (6,531) Q
  531 FORMAT (7X,-HEAT GENERATED PER CYCLE -,45(1H.),-Q = -,1PE10.4,1X,-
     C(IN-LBF)-)
      GO TO 148
  200 STOP
      END
-MAP,S
-XQT




4.71238 1.4      28.8     4.44     0.5      -1.5     280.0    168.0    1.0
155.0       1.0
155.0       1.2
165.0       1.05
175.0       1.05
-1.0
-FIN
```

APPENDIX B

STABILITY PROGRAM

The stability program is included as a design tool, and is reproduced in FORTRAN IV language on the following pages.

```
-RUN FORTN,01E25602,HORN-W-A,1,100
-FOR,IS MAIN
C       THIS PROGRAM FINDS AMPLITUDE AND FREQUENCY VARIATION DUE TO
C       DAMPING OTHER THAN DESIGN VALUE, IF ENGAGEMENT AND DISENGAGEMENT
C       ARE DETERMINED FROM LOAD ANGULAR POSITION.
C
        IMPLICIT REAL (A-Z)
        INTEGER I,L
        INTEGER IC
        DIMENSION TIME(500),THETA(500),PHID(500),TTWO(25)
        DIMENSION MCLUT(500),TCLUT(500)
        COMMON T1,THETA1,THETA2,OMEGAN,OMEGAD,ZETAWN,ZETA,RADICL,BETA1
        COMMON BETA1D,BETA2,BETA2D,TAU,TONE(25),TR,K
        COMMON /A/ L
        COMMON /B/ TQ,ALPHA,J,R,PHI1D0,PHI2D0,GAMA1D(25),GAMA2D(25)
        COMMON /C/ TTHREE(25),C,CDSIGN
  888   READ (5,100,END=200) T1,THETA1,THETA2,TAU,TR,K,JSUBT,CAPTHE
        IF (T1 .LT. 0.0) GO TO 200
  100   FORMAT (8F9.0)
        READ (5,102) TQ,ALPHA,J,R
  102   FORMAT (4F10.0)
        READ (5,101) C,CDSIGN
  101   FORMAT (2F10.0)
        READ (5,555) IC
  555   FORMAT (I1)
        WRITE (6,500)
  500   FORMAT (1H1,5X,-T1-,6X,-THETA1-,3X,-THETA2-,6X,-TAU-,8X,-TR-,9X,-K
       C-,6X,-JSUBT-)
        WRITE (6,501) T1,THETA1,THETA2,TAU,TR,K,JSUBT
  501   FORMAT (7F10.4)
        WRITE (6,502)
  502   FORMAT (//,2X,-CAPTHE-,7X,-TQ-,6X,-ALPHA-,7X,-J-,10X,-R-,8X,-C-,6X
       C,-CDSIGN-)
        WRITE (6,503) CAPTHE,TQ,ALPHA,J,R,C,CDSIGN
  503   FORMAT (7F10.4)
        ZETA = C / (2.0 * SQRT(JSUBT * K))
        OMEGAN = SQRT(K / JSUBT)
        OMEGAD = OMEGAN * SQRT(1.0 - ZETA**2)
        ZETAWN = ZETA * OMEGAN
        RADICL = SQRT(1.0 - ZETA**2)
        THE10 = CAPTHE
        THE1D0 = 0.0
        PHI1D0 = -TQ / ALPHA
        L = 1
        SUM = 0.0
        TTHREE(L) = 0.0
        IF  (IC .EQ. 2) GO TO 444
        TONE(L) = WHEN1(THE10,THE1D0)
        THE20 = BETA1
        THE2D0 = BETA1D
        PHI2D0 = GAMA1D(L)
  777   TTWO(L) = WHEN2(THE20,THE2D0)
        DO 1 I=1,500
        TIME(I) = I - 1
        TIME(I) = TIME(I) * 0.02
        TOTIME = TIME(I)
  216   TIME(I) = TOTIME - SUM
        IF (TIME(I) .GT. TONE(L)) GO TO 205
        ARG1 = OMEGAD * TIME(I)
        THETA(I) = EXP(-ZETAWN * TIME(I)) * (THE10 * (COS(ARG1) + ZETA / R
```

```
   A ADICL * SIN(ARG1)) + THE1D0 * (1.0 / OMEGAD * SIN(ARG1)))
      PHID(I) = -TQ/ALPHA + (TQ / ALPHA + PHI1D0) * EXP(ALPHA / J * TIM
   A E(I))
      MCLUT(I) = PHID(I) / R * (60.0 / 6.28319)
      TCLUT(I) = (EXP(-ZETAWN * TIME(I)) * (THE10 * OMEGAD * (ZETA / RAD
   A ICL * COS(ARG1) - SIN(ARG1)) + THE1D0 * COS(ARG1)) - ZETAWN *
   B (EXP(-ZETAWN * TIME(I)) * (THE10 * (COS(ARG1) + ZETA / RADICL * S
   C IN(ARG1)) + THE1D0 * (1.0 / OMEGAD * SIN(ARG1))))) * (60.0 / 6.28
   D319)
      GO TO 217
205 IF (TIME(I) .GT. TTWO(L) ) GO TO 215
      TIME(I) = TIME(I) - TONE(L)
      ARG2 = OMEGAD * TIME(I)
      THETA(I) = EXP(-ZETAWN * TIME(I)) * (THE20 * (COS(ARG2) + ZETA / R
   A ADICL * SIN(ARG2)) + THE2D0 * (1.0 / OMEGAD * SIN(ARG2))) + TR /
   B K * (TIME(I) - 2.0 * ZETA / OMEGAN * (1.0 - EXP(-ZETAWN * TIME(I)
   C ) * (COS(ARG2) + (ZETA - 1.0 / (2.0 * ZETA)) / RADICL * SIN(ARG2)
   D )))
      PHID(I) = (PHI2D0 + TQ / ALPHA - (TR * J) / (R * ALPHA**2)) * EXP(
   A ALPHA / J * TIME(I)) + TR / (R * ALPHA) * TIME(I) + (TR * J / (R*
   B ALPHA**2) - TQ / ALPHA)
      MCLUT(I) = PHID(I) / R * (60.0 / 6.28319)
      TCLUT(I) = (EXP(-ZETAWN * TIME(I)) * (THE20 * OMEGAD * (ZETA / RAD
   A ICL * COS(ARG2) - SIN(ARG2)) + THE2D0 * COS(ARG2)) - ZETAWN *
   B           (EXP(-ZETAWN * TIME(I)) * (THE20 * (COS(ARG2) + ZETA / R
   C ADICL * SIN(ARG2)) + THE2D0 * (1.0 / OMEGAD * SIN(ARG2))))
   D + TR / K * (1.0 + (2.0 * ZETA / OMEGAN * EXP(-ZETAWN * TIME(I)) *
   E (OMEGAD * ((ZETA - 1.0 / (2.0 * ZETA)) / RADICL * COS(ARG2) - SIN
   F (ARG2))) - 2.0 * ZETA**2 * EXP(-ZETAWN * TIME(I)) * (COS(ARG2) +
   G (ZETA - 1.0 / (2.0 * ZETA)) / RADICL * SIN(ARG2))))) * (60.0 /
   H 6.28319)
      TIME(I) = TIME(I) + TONE(L)
      GO TO 217
215 SUM = SUM + TTWO(L)
      THE10 = BETA2
      THE1D0 = BETA2D
      PHI1D0 = GAMA2D(L)
      L = L + 1
      TTHREE(L) = WHEN3(THE10,THE1D0)
      TONE(L) = WHEN1(THE10,THE1D0)
      THE20 = BETA1
      THE2D0 = BETA1D
      PHI2D0 = GAMA1D(L)
      TTWO(L) = WHEN2(THE20,THE2D0)
      GO TO 216
444 THE10 = 0.0
      TONE(L) = 0.0
      THE20 = 0.0
      THE2D0 = 0.0
      PHI2D0 = - TQ / ALPHA
      GO TO 777
217 THETA(I) = THETA(I) * 57.2958
      PHID(I) = PHID(I) * (60.0 / 6.28319)
      TIME(I) = TOTIME
    1 CONTINUE
      PERA = (TTWO(2) + TTWO(3) + TTWO(4)) / 3.0
      TRANSA = ((TTWO(2) - TONE(2)) + (TTWO(3) - TONE(3)) + (TTWO(4) - T
   C ONE(4))) / 3.0
      ACCELA = PERA - TRANSA
      WRITE (6,504) ZETA
```

```
  504 FORMAT (//,1X,-ZETA = -,F5.3)
      WRITE (6,505) PERA
  505 FORMAT (/,1X,-AVERAGE PERIOD = -,F7.4,1X,-(SEC)-)
      WRITE (6,506) TRANSA
  506 FORMAT (1X,-AVERAGE TRANSMISSION INTERVAL = -,F7.4,1X,-(SEC)-)
      WRITE (6,507) ACCELA
  507 FORMAT (1X,-AVERAGE ACCELERATION INTERVAL = -,F7.4,1X,-(SEC)-)
      T1A = (TONE(2) + TONE(3) + TONE(4)) / 3.0
      T2A = (TTWO(2) + TTWO(3) + TTWO(4)) / 3.0
      T3A = (TTHREE(2) + TTHREE(3) + TTHREE(4)) / 3.0
      T1 = T1A - T3A
      T2 = T2A - T3A
      T3 = T1A + (T2A - T3A)
      OMEG1 =(GAMA1D(2) + GAMA1D(3) + GAMA1D(4)) / 3.0
      OMEG2 = (GAMA2D(2) + GAMA2D(3) + GAMA2D(4)) / 3.0
      OMEG1P = OMEG1 * (60.0 / 6.28319)
      OMEG2P = OMEG2 * (60.0 / 6.28319)
      WRITE (6,509) OMEG1P
  509 FORMAT (//,1X,-AVERAGE OMEGA1 = -,F10.2,1X,-(RPM)-)
      WRITE (6,510) OMEG2P
  510 FORMAT (1X,-AVERAGE OMEGA2 = -,F10.2,1X,-(RPM)-)
C     COMPUTE WORK
      A1 = - (TQ / ALPHA + OMEG2)
      B1 = - TQ / ALPHA
      C1 = OMEG1 + TQ / ALPHA - (TR * J) / (R * ALPHA**2)
      D1 = TR / (R * ALPHA)
      E1 = (TR * J) / (R * ALPHA**2) - TQ / ALPHA
      F1 = (TR * J) / (R * ALPHA**2)
      WORK = - ALPHA * (C1**2 * (J / (2.0 * -ALPHA)) * (EXP((2.0 * ALPHA
     A ) / J * T2) - EXP((2.0 * ALPHA) / J * T1)) + 2.0 * C1 * D1 * (J /
     B ALPHA)**2 * ((1.0 - ALPHA / J * T2) * EXP(ALPHA / J * T2) - (1.0
     C - ALPHA / J * T1) * EXP(ALPHA / J * T1)) + C1 * (E1 + F1) * (J /
     D - ALPHA) * (EXP(ALPHA / J * T2) - EXP(ALPHA / J * T1)) + 1.0 / 3.
     E * D1**2 * (T1**3 - T2**3) + 0.5 * D1 * (E1 + F1) * (T1**2 - T2**2
     F ) + E1 * F1 * (T1 - T2)) + (TQ * B1 + ALPHA * B1**2) * (T3 - T2)
     G - (TQ * A1 * J / ALPHA + 2.0 * B1 * A1 * J) * (EXP(ALPHA / J * T3
     H ) - EXP(ALPHA / J * T2)) + A1**2 * J / 2.0 * (EXP(2.0 * ALPHA / J
     I * T3) - EXP(2.0 * ALPHA / J * T2))
C
C     COMPUTE AVERAGE HORSEPOWER
      HPWR = WORK * (1.0 / 12.0) * (1.0 /PERA) * (1.0 / 550.0)
      WRITE (6,508) HPWR
  508 FORMAT (//,1X,-AVERAGE HORSEPOWER = -,F6.4,///)
      WRITE (6,104)
  104 FORMAT (1X,-TIME-,6X,-THETA-,6X,-OMEGA-,12X,-OMEGA/R-,3X,-TUBVEL-)
      WRITE (6,105)
  105 FORMAT (1X,-(SEC)-,5X,-(DEG)-,6X,-(RPM)-,13X,-(RPM)-,5X,-(RPM)-,/)
      DO 2 I=1,500
    2 WRITE (6,103) TIME(I),THETA(I),PHID(I),MCLUT(I),TCLUT(I)
  103 FORMAT (F6.2,2F10.2,10X,2F10.2)
      GO TO 888
  200 STOP
      END
```

```
-FOR,IS WHEN1
      FUNCTION WHEN1(THE10,THE1D0)
      IMPLICIT REAL (A-Z)
      INTEGER M
      INTEGER L
      DIMENSION TM(25),THET(25)
      COMMON T1,THETA1,THETA2,OMEGAN,OMEGAD,ZETAWN,ZETA,RADICL,BETA1
      COMMON BETA1D,BETA2,BETA2D,TAU,TONE(25),TR,K
      COMMON /A/ L
      COMMON /B/ TQ,ALPHA,J,R,PHI1D0,PHI2D0,GAMA1D(25),GAMA2D(25)
      COMMON /C/ TTHREE(25),C,CDSIGN
C
      IF (C .EQ. CDSIGN) GO TO 241
      DO 4 M = 1,25
      W = M
      TM(M) = 0.24 + W * 0.02
      ARG1 = OMEGAD * TM(M)
      THET(M)  = EXP(-ZETAWN * TM(M)   ) * (THE10 * (COS(ARG1) + ZETA / R
     A ADICL * SIN(ARG1)) + THE1D0 * (1.0 / OMEGAD * SIN(ARG1)))
      IF (THET(M) .LT. THETA1) GO TO 318
    4 CONTINUE
      WRITE (6,319)
  319 FORMAT (1X,-M=25 IN WHEN1-)
  241 TEE1S = 0.1
      TEE1L = TTHREE(L) + T1
      TEE1 = TEE1L
      GO TO 202
  318 TEE1L = TM(M)
      TEE1S = 0.2
      TEE1 = TEE1L
  202 ARG1 = OMEGAD * TEE1
      BETA1    = EXP(-ZETAWN * TEE1    ) * (THE10 * (COS(ARG1) + ZETA / R
     A ADICL * SIN(ARG1)) + THE1D0 * (1.0 / OMEGAD * SIN(ARG1)))
      IF (ABS(BETA1 - THETA1) .LT. 1.E-2) GO TO 203
      IF (THETA1 - BETA1 .LT. 0.0) TEE1S = TEE1
      IF (THETA1 - BETA1 .GT. 0.0) TEE1L = TEE1
      TEE1 = (TEE1S + TEE1L) / 2.0
      GO TO 202
  203 WHEN1 = TEE1
      BETA1D = EXP(-ZETAWN * TEE1) * (THE10 * OMEGAD * (ZETA / RADICL *
     A COS(ARG1) - SIN(ARG1)) + THE1D0 * COS(ARG1)) - ZETAWN * BETA1
      GAMA1D(L)=-TQ/ALPHA + (TQ / ALPHA + PHI1D0) * EXP(ALPHA / J *TEE1)
      RETURN
      END
```

```
-FOR,IS WHEN2
      FUNCTION WHEN2(THE20,THE2D0)
      IMPLICIT REAL (A-Z)
      INTEGER N
      INTEGER L
      DIMENSION TMM(30),THE(30)
      COMMON TI,THETA1,THETA2,OMEGAN,OMEGAD,ZETAWN,ZETA,RADICL,BETA1
      COMMON BETA1D,BETA2,BETA2D,TAU,TONE(25),TR,K
      COMMON /A/ L
      COMMON /B/ TQ,ALPHA,J,R,PHI1D0,PHI2D0,GAMA1D(25),GAMA2D(25)
      DO 5 N = 1,30
      X = N
      TMM(N) = (TONE(L) + 0.3) + X * 0.02
      TMM(N) = TMM(N) - TONE(L)
      ARG2 = OMEGAD * TMM(N)
      THE(N)    = EXP(-ZETAWN * TMM(N) ) * (THE20 * (COS(ARG2) + ZETA / R
     A ADICL * SIN(ARG2)) + THE2D0 * (1.0 / OMEGAD * SIN(ARG2))) + TR /
     B K * (TMM(N)  - 2.0 * ZETA / OMEGAN * (1.0 - EXP(-ZETAWN * TMM(N)
     C ) * (COS(ARG2) + (ZETA - 1.0 / (2.0 * ZETA)) / RADICL * SIN(ARG2)
     D )))
      TMM(N) = TMM(N) + TONE(L)
      IF (THE(N) .GT. THETA2) GO TO 321
    5 CONTINUE
      WRITE (6,322)
  322 FORMAT (1X,-N=30 IN WHEN2-)
  321 TEE2L = TMM(N)
      TEE2S = TONE(L) + 0.3
      TEE2 = TEE2S
  206 TEE2 = TEE2 - TONE(L)
      ARG2 = OMEGAD * TEE2
      BETA2    = EXP(-ZETAWN * TEE2   ) * (THE20 * (COS(ARG2) + ZETA / R
     A ADICL * SIN(ARG2)) + THE2D0 * (1.0 / OMEGAD * SIN(ARG2))) + TR /
     B K * (TEE2    - 2.0 * ZETA / OMEGAN * (1.0 - EXP(-ZETAWN * TEE2
     C ) * (COS(ARG2) + (ZETA - 1.0 / (2.0 * ZETA)) / RADICL * SIN(ARG2)
     D )))
      TEE2 = TEE2 + TONE(L)
      IF (ABS(BETA2- THETA2) .LT. 1.E-2) GO TO 207
      IF (BETA2- THETA2 .LT. 0.0) TEE2S = TEE2
      IF (BETA2 -THETA2 .GT. 0.0) TEE2L = TEE2
      TEE2 = (TEE2S + TEE2L) / 2.0
      GO TO 206
  207 WHEN2 = TEE2
      TEE2 = TEE2 - TONE(L)
      BETA2D = EXP(-ZETAWN * TEE2) * (THE20 * OMEGAD * (ZETA / RADICL *
     A COS(ARG2) - SIN(ARG2)) + THE2D0 * COS(ARG2)) - ZETAWN *
     Z          (EXP(-ZETAWN * TEE2   ) * (THE20 * (COS(ARG2) + ZETA / R
     Z ADICL * SIN(ARG2)) + THE2D0 * (1.0 / OMEGAD * SIN(ARG2))))
     B+TR / K * (1.0 + (2.0 * ZETA / OMEGAN * EXP(-ZETAWN * TEE2) *(OMEG
     C AD * ((ZETA - 1.0 / (2.0 * ZETA)) / RADICL * COS(ARG2) - SIN(ARG2
     D ))) - 2.0 * ZETA**2 * EXP(-ZETAWN * TEE2) * (COS(ARG2) + (ZETA -
     E 1.0 / (2.0 * ZETA)) / RADICL * SIN(ARG2))))
      GAMA2D(L)=(PHI2D0 + TQ / ALPHA - (TR * J) / (R * ALPHA**2)) * EXP(
     A ALPHA / J * TEE2    ) + TR / (R * ALPHA) * TEE2    + (TR * J / (R*
     B ALPHA**2) - TQ / ALPHA)
      TEE2 = TEE2 + TONE(L)
      RETURN
      END
```

```
-FOR,IS WHEN3
      FUNCTION WHEN3(THE10,THE1D0)
      IMPLICIT REAL (A-Z)
      INTEGER L
      COMMON T1,THETA1,THETA2,OMEGAN,OMEGAD,ZETAWN,ZETA,RADICL,BETA1
      COMMON BETA1D,BETA2,BETA2D,TAU,TONE(25),TR,K
      COMMON /A/ L
      COMMON /B/ TQ,ALPHA,J,R,PHI1D0,PHI2D0,GAMA1D(25),GAMA2D(25)
C
      TEE3S = 0.0
      TEE3L = T1 / 2.0
      TEE3 = TEE3S
  519 ARG1 = OMEGAD * TEE3
      PHI1D  = EXP(-ZETAWN * TEE3) * (THE10 * OMEGAD * (ZETA / RADICL *
     A COS(ARG1) - SIN(ARG1)) + THE1D0 * COS(ARG1)) - ZETAWN *
     B          (EXP(-ZETAWN * TEE3    ) * (THE10 * (COS(ARG1) + ZETA / R
     C ADICL * SIN(ARG1)) + THE1D0 * (1.0 / OMEGAD * SIN(ARG1))))
      DISPL = EXP(-ZETAWN * TEE3) * (THE10 * (COS(ARG1) + ZETA / RADICL
     A * SIN(ARG1)) + THE1D0 * (1.0 / OMEGAD * SIN(ARG1)))
      IF (ABS(PHI1D - 0.0) .LT. 1.E-2 .AND. (DISPL .GT. 0.0)) GO TO 520
      IF (PHI1D .GT. 0.0) TEE3S = TEE3
      IF (PHI1D .LT. 0.0) TEE3L = TEE3
      TEE3 = (TEE3S + TEE3L) / 2.0
      GO TO 519
  520 WHEN3 = TEE3
      RETURN
      END
-XQT




0.6361    -.53      3.99      1.4       1179.69  155.0     4.44      4.18
280.0       -1.5      1.0         19.88
25.0        28.8
1

0.6361    -.53      3.99      1.4       1179.69  155.0     4.44      4.18
280.0       -1.5      1.0         19.88
28.8        28.8
2
-1.0
-FIN
```

APPENDIX C

SUPPLEMENTARY DATA

On the following pages are additional plots and discussion deemed helpful in clarifying certain aspects of the research presented in the main text. As will be noticed, the format of this data varies somewhat from that presented earlier. This discussion represents steps taken throughout the work and as such, generally prompted some change in the program or in the design parameters. Therefore, the reader is cautioned to examine carefully any notes accompanying plots and be aware of changes made. The bulk of this data was obtained from a hypothetical motor derived from the NEMA standards for single phase induction motors. A particular motor was not selected until the final runs. Values obtained for a typical .5 Hp motor were

$$\alpha = -2.28 \quad \text{in-lbf-sec/rad}$$

$$TQ = 431 \quad \text{in-lbf}$$

In the following material the data is for this motor unless specified otherwise.

The decision to introduce a gear reduction was prompted as follows. Early results indicated that the mechanism would function with the motor operating at low speed and with high torque. As Figure 43 shows, this involves operation considerably in excess of breakdown for a real motor. The problem is that the linear torque v.s. speed relation is invalid past breakdown but that the computer is not programmed to acknowledge this fact. Thus, solutions were obtained in the above range.

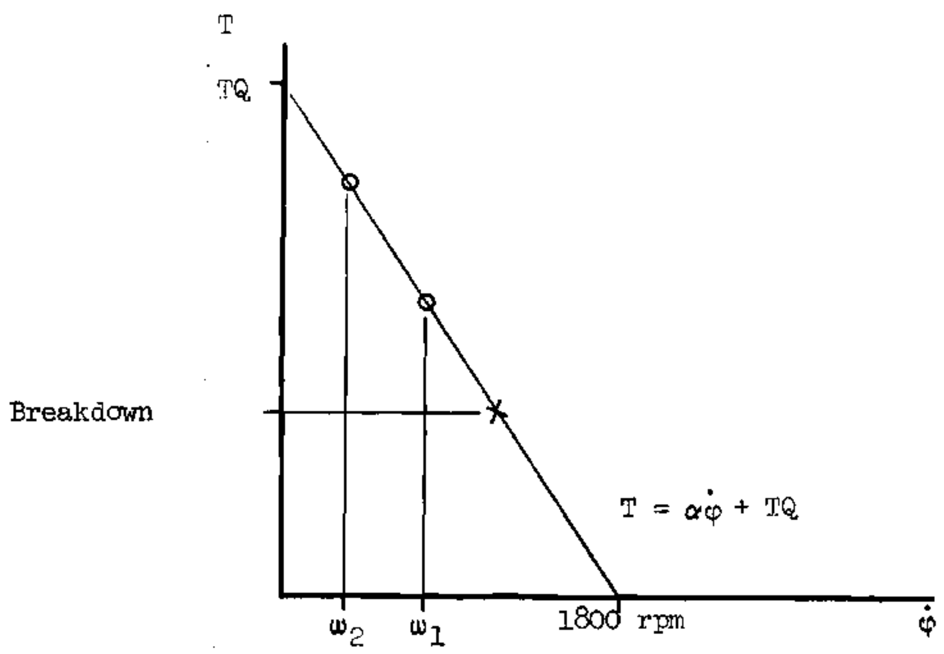


Figure 43. Operating in Excess of Breakdown Torque

The data indicated that if a new motor were used which developed high torque at low speed then a successful solution would result. A "new" motor having the above characteristics may be constructed by adding a gear reduction to the present motor. If this motor-gear train unit were enclosed in a "black-box" with only the output shaft emerging, then the box appears to contain a motor with the desired characteristics.

Once the gear reduction is included, a new design variable, the reduction ratio R, becomes evident. If the flywheel inertia is also allowed to vary then a map of useful inertia-gear ratio combinations may be obtained. A map of this sort is intended to portray regions of satisfactory designs where the criteria are:

(1) The motor speed must not drop below breakdown speed.

(2) The drive side of the clutch must rotate faster than the load side at all times to transmit torque.

(3) The motor power loading may not exceed the design value. For the hypothetical motor, a map such as is shown in Figure 44 was obtained for a fixed value of K. Only those combinations at J_F and R which determine a point on the shaded side of the curves meet all three criteria. It can be seen that rather large values of both parameters are called for. In fact, due to practical construction limitations, as discussed in the text, J_F was arbitrarily limited to a value of 1.0 in succeeding work although some adjustment of this value is possible. Also, it is desirable to keep the reduction ratio as low as possible due to cost and complexity considerations. Reflected in this same map is the desirability of reducing the slip which in turn reduces the power requirement. This sort of plot

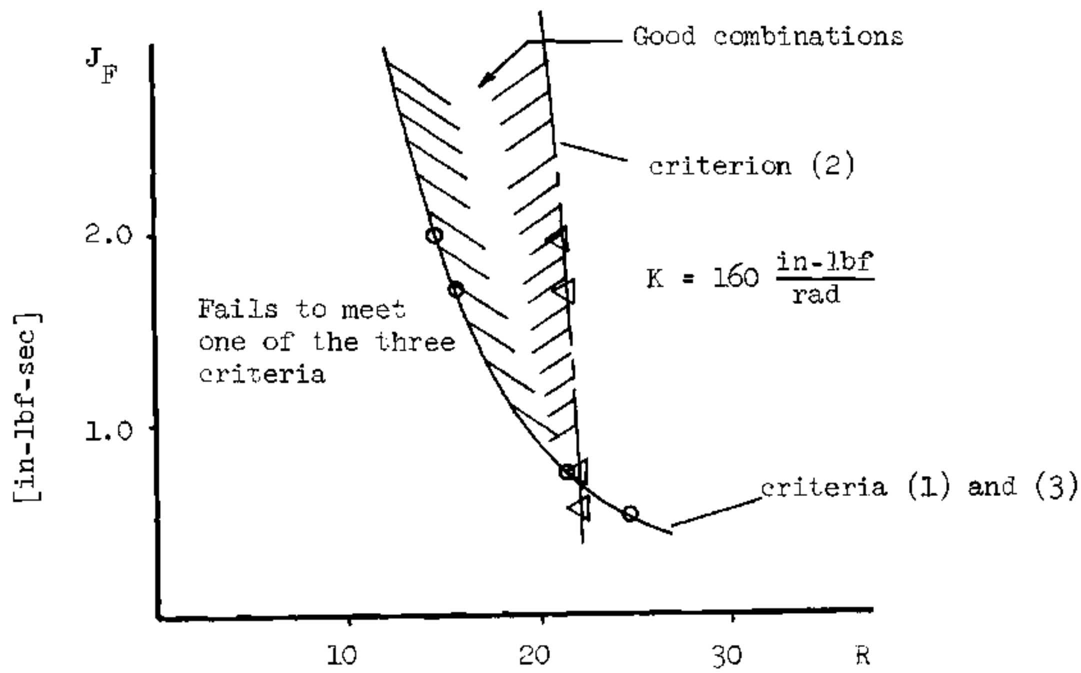


Figure 44. Map of Successful J_F and R Combinations

indicates the complexity involved in meeting all the requirements of a complex system, and justifies certain design restrictions.

To further clarify why J_F was fixed and R left as a variable the following plot was constructed, Figure 45. The curves indicate that motor workload is much more responsive to changes in R than to changes in J_F. The three curves represent a total variation of more than doubling the upper most value of J_F. An equivalent reduction in work may be obtained by only increasing R by ten percent. An interesting feature of this curve is the intersection area below which an increase in J_F yields an <u>increase</u> in motor work. This phenomenon was

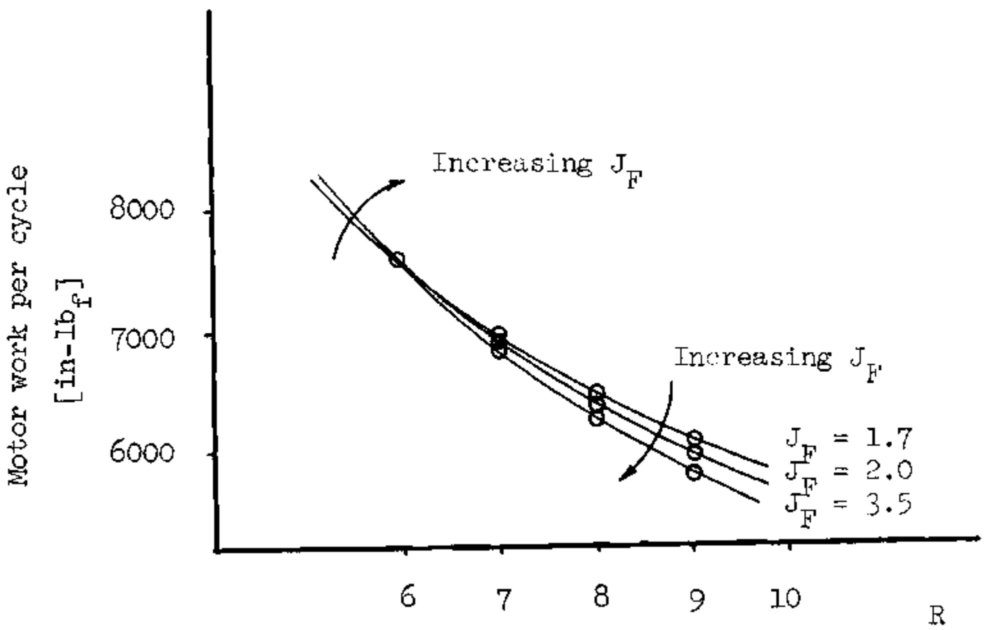


Figure 45. Effect of Varying R and J_F on Motor Workload.

not investigated since low values of R were unacceptable due to motor overload.

The effect of varying K, though extremely complex in theory, is illustrated in Figure 46 as obtained from the program in its final form. It can be seen that the motor power loading and the appropriate gear ratio for minimum slip obey simple, nearly inverse relationships with the spring constant.

The variation of clutch operating temperature with percent slip is illustrated in Figure 47. Thus, for no slip, the equilibrium temperature is merely the ambient, and for increasing slip the ultimate temperature rises sharply.

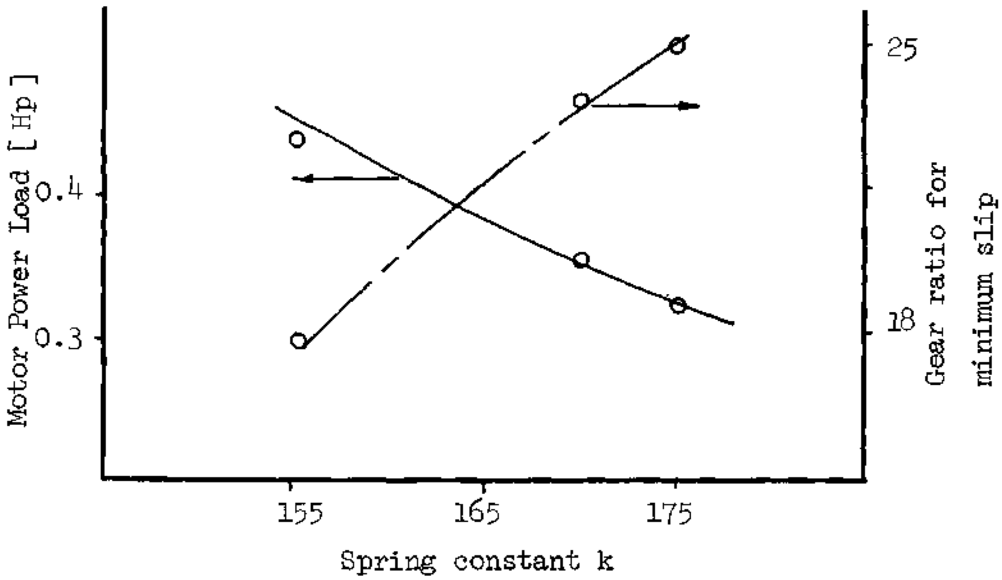


Figure 46. Effect of Varying k on Motor Power Load

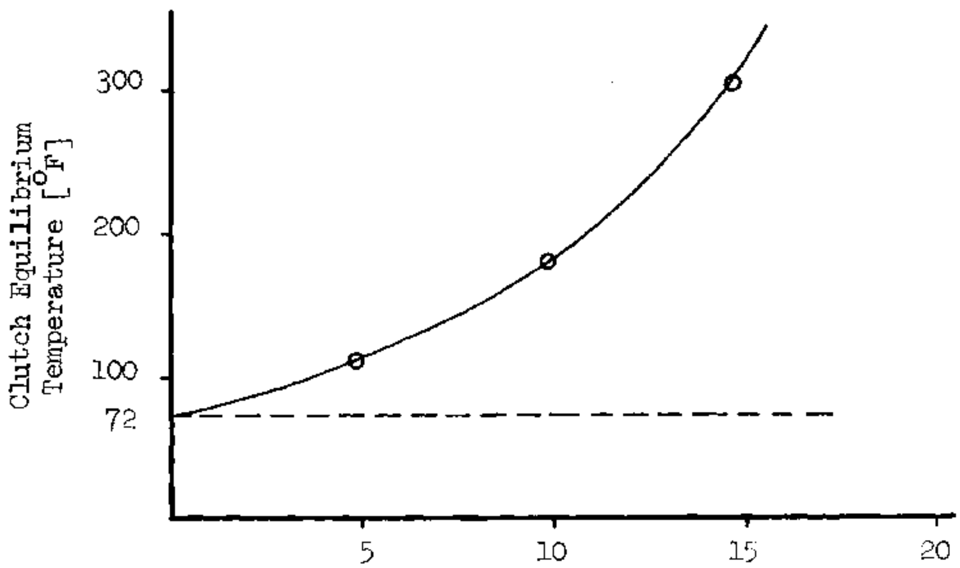


Figure 47. Equilibrium Clutch Temperature

The bulk of these curves are intended to indi ate trends only and should be interpreted as design guidelines. Finally, reproductions of additional computer output are included for reference.

```
GIVEN-                                                            DATA-
     DESIRED ARC ---------------------------------------------------ARC =     4.7124 (RAD)
     DESIRED PERIOD ------------------------------------------------TAU =  1.40 (SEC)
     MAXIMUM LOAD DAMPING COEFFICIENT (INCLUDES
        BEARINGS,ETC. ON LOAD SIDE OF CLUTCH) ----------------------C =    28.8000 (IN-LBF-SEC)
     MAXIMUM TOTAL LOAD INERTIA (INCLUDES SHAFT
        AND LOAD SIDE OF CLUTCH) -----------------------------------JSUBT =    4.4400 (IN-LBF-SEC-SEC)
     ALLOWABLE HORSEPOWER LOAD ON MOTOR ----------------------------HP =  .500
     MOTOR PERFORMANCE CURVE - SLOPE -------------------------------ALPHA =    -2.2000 (IN-LBF-SEC/RAD)
                               INTERCEPT ---------------------------TQ =  431.00 (IN-LBF)
     MOTOR BREAKDOWN SPEED -----------------------------------------BRKDWN =  168.00 (RAD/SEC)
     FLYWHEEL INERTIA (CONSIDERS GEAR,PINION,
        AND MOTOR SIDE OF CLUTCH MASSLESS) -------------------------J =   1.0000 (IN-LBF-SEC-SEC)

FIND-                                                             RESULTS-
     TORSIONAL SPRING CONSTANT -------------------------------------K =   141.12 (IN-LBF/RAD)
     STEP-DOWN GEAR RATIO ------------------------------------------R = 17.53
     RAMP TORQUE SLOPE ---------------------------------------------TR =  1289.00 (IN-LBF/SEC)
     TIME TO ENGAGE CLUTCH -----------------------------------------T1 =    .6810 (SEC)
     TIME TO DISENGAGE CLUTCH --------------------------------------T2 =   1.3571 (SEC)
     POSITION TO ENGAGE CLUTCH -------------------------------------THETA1 =      -.47 (RAD)
     POSITION TO DISENGAGE CLUTCH ----------------------------------THETA2 =      3.95 (RAD)
     LOAD ANGULAR VELOCITY AT DISENGAGEMENT ------------------------THET2D =    10.17 (RAD/SEC)
     MOTOR SPEED FLUCTUATION- HIGH ---------------------------------OMEGA1 =   167.09 (RAD/SEC)
                              LOW ----------------------------------OMEGA2 =   170.42 (RAD/SEC)
     DAMPING RATIO -------------------------------------------------ZETA =  .58
     PEAK TORQUE TRANSMITTED ---------------------------------------TQMAX =   845.43 (IN-LBF)
     TIME INTERVAL FOR TRANSMISSION --------------------------------TRANS =    .6559 (SEC)
     TIME INTERVAL FOR ACCELERATING --------------------------------ACCEL =    .7441 (SEC)
     WORK PER CYCLE DONE BY MOTOR ----------------------------------WORK =  5127.02 (IN-LBF)
     AVERAGE HORSEPOWER --------------------------------------------HPWR =  .5549
     INITIAL ANGULAR DISPLACEMENT ----------------------------------CAPTHE =      4.25 (RAD)
```

```
GIVEN-                                                          DATA-
     DESIRED ARC ------------------------------------------------ARC =     4.7124 (RAD)
     DESIRED PERIOD ---------------------------------------------TAU =  1.40 (SEC)
     MAXIMUM LOAD DAMPING COEFFICIENT (INCLUDES
         BEARINGS,ETC. ON LOAD SIDE OF CLUTCH) ------------------C =     28.8000 (IN-LBF-SEC)
     MAXIMUM TOTAL LOAD INERTIA (INCLUDES SHAFT
         AND LOAD SIDE OF CLUTCH) -------------------------------JSUBT =    4.4400 (IN-LBF-SEC-SEC)
     ALLOWABLE HORSEPOWER LOAD ON MOTOR -------------------------HP =  .500
     MOTOR PERFORMANCE CURVE - SLOPE ----------------------------ALPHA =     -2.2800 (IN-LBF-SEC/RAD)
                               INTERCEPT ------------------------TO =   431.00 (IN-LBF)
     MOTOR BREAKDOWN SPEED --------------------------------------DRKDWN =   168.00 (RAD/SEC)
     FLYWHEEL INERTIA (CONSIDERS GEAR,PINION,
         AND MOTOR SIDE OF CLUTCH MASSLESS) ---------------------J =   1.0000 (IN-LBF-SEC-SEC)


FINDS-                                                          RESULTS-
     TORSIONAL SPRING CONSTANT -------------------------------------K =   155.00 (IN-LBF/RAD)
     STEP DOWN GEAR RATIO ------------------------------------------R =  6.33
     RAMP TORQUE SLOPE ---------------------------------------------TR =  1179.69 (IN-LBF/SEC)
     TIME TO ENGAGE CLUTCH -----------------------------------------T1 =    .6361 (SEC)
     TIME TO DISENGAGE CLUTCH --------------------------------------T2 =   1.3513 (SEC)
     POSITION TO ENGAGE CLUTCH -------------------------------------THETA1 =     4.53 (RAD)
     POSITION TO DISENGAGE CLUTCH ----------------------------------THETA2 =     3.99 (RAD)
     LOAD ANGULAR VELOCITY AT DISENGAGEMENT ------------------------THET2D =     8.35 (RAD/SEC)
     MOTOR SPEED FLUCTUATION- HIGH ---------------------------------OMEGA1 =   182.55 (RAD/SEC)
                              LOW ----------------------------------OMEGA2 =   158.12 (RAD/SEC)
     DAMPING RATIO -------------------------------------------------ZETA =  .55
     PEAK TORQUE TRANSMITTED ---------------------------------------TQMAX =   843.75 (IN-LBF)
     TIME INTERVAL FOR TRANSMISSION --------------------------------TRANS =    .7152 (SEC)
     TIME INTERVAL FOR ACCELERATING --------------------------------ACCEL =    .6848 (SEC)
     WORK PER CYCLE DONE BY MOTOR ----------------------------------WORK = 11238.44 (IN-LBF)
     AVERAGE HORSEPOWER --------------------------------------------HPWR = 1.2163
     INITIAL ANGULAR DISPLACEMENT ----------------------------------CAPTHE =     4.18 (RAD)
```

```
GIVEN-                                                                     DATA-
      DESIRED ARC ---------------------------------------------------------ARC =      4.7124 (RAD)
      DESIRED PERIOD ------------------------------------------------------TAU =  1.40 (SEC)
      MAXIMUM LOAD DAMPING COEFFICIENT (INCLUDES
           BEARINGS,ETC. ON LOAD SIDE OF CLUTCH) --------------------------C =     28.8000 (IN-LBF-SEC)
      MAXIMUM TOTAL LOAD INERTIA (INCLUDES SHAFT
           AND LOAD SIDE OF CLUTCH) ---------------------------------------JSUBT =   4.4400 (IN-LBF-SEC-SEC)
      ALLOWABLE HORSEPOWER LOAD ON MOTOR ----------------------------------HP =  .500
      MOTOR PERFORMANCE CURVE - SLOPE -------------------------------------ALPHA =    -2.2000 (IN-LBF-SEC/RAD)
                                INTERCEPT ---------------------------------TQ =   431.00 (IN-LBF)
      MOTOR BREAKDOWN SPEED -----------------------------------------------BRKDWN =   168.00 (RAD/SEC)
      FLYWHEEL INERTIA (CONSIDERS GEAR,PINION,
           AND MOTOR SIDE OF CLUTCH MASSLESS) -----------------------------J =   1.0000 (IN-LBF-SEC-SEC)


FINDS-                                                                   RESULTS-
      TORSIONAL SPRING CONSTANT -------------------------------------------K =   170.00 (IN-LBF/RAD)
      STEP DOWN GEAR RATIO ------------------------------------------------R = 25.95
      RAMP TORQUE SLOPE ---------------------------------------------------TR =  1128.91 (IN-LBF/SEC)
      TIME TO ENGAGE CLUTCH -----------------------------------------------T1 =      .5962 (SEC)
      TIME TO DISENGAGE CLUTCH --------------------------------------------T2 =   1.3606 (SEC)
      POSITION TO ENGAGE CLUTCH -------------------------------------------THETA1 =      -.60 (RAD)
      POSITION TO DISENGAGE CLUTCH ----------------------------------------THETA2 =      5.99 (RAD)
      LOAD ANGULAR VELOCITY AT DISENGAGEMENT ------------------------------THET2D =      6.98 (RAD/SEC)
      MOTOR SPEED FLUCTUATION- HIGH ---------------------------------------OMEGA1 =   167.15 (RAD/SEC)
                               LOW ----------------------------------------OMEGA2 =   161.02 (RAD/SEC)
      DAMPING RATIO -------------------------------------------------------ZETA =  .52
      PEAK TORQUE TRANSMITTED ---------------------------------------------TQMAX =    863.17 (IN-LBF)
      TIME INTERVAL FOR TRANSMISSION --------------------------------------TRANS =     .7646 (SEC)
      TIME INTERVAL FOR ACCELERATING --------------------------------------ACCEL =     .6354 (SEC)
      WORK PER CYCLE DONE BY MOTOR ----------------------------------------WORK =  3601.65 (IN-LBF)
      AVERAGE HORSEPOWER --------------------------------------------------HPWR =  .3896
      INITIAL ANGULAR DISPLACEMENT ----------------------------------------CAPTHE =     4.12 (RAD)
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BIBLIOGRAPHY

Literature Cited

1. R. Anderson, "Mechanical-Contact Clutches for Space Applications," Masters Thesis, Georgia Institute of Technology, 1966.

2. F. Scheid, Theory and Problems of Numerical Analysis, McGraw-Hill Book Company, Inc., New York, 1968.

3. J. E. Shigley, Mechanical Engineering Design, McGraw-Hill Book Company, Inc., New York, 1963.

4. A. F. Gagne, Jr., "Torque Capacity and Design of Cone and Disk Clutches," Machine Design, Vol. 24, No. 12, pp. 182-187, December, 1953.

5. G. A. G. Fazekaz, "Temperature Gradient and Heat Stress in Brake Drums," SAE Transactions, 61, 279, 1953.

Other References

Breimeier, P. H., "Fractional-Horsepower Induction Motors," Machine Design, Vol. 42, No. 9, April, 1970.

Finkin, E. F., "Equations for the Wear and Contact Pressure of Annular Composite Electromagnetic Clutch Rotors," Journal of Lubrication Technology, Trans. ASME, V. 92, Ser. F., N. 2, April, 1970.

Jania, Z. J., "Friction Clutch Transmissions," Machine Design, V. 30, N. 25, December 11, 1958.

Johnson, R. C., Optimum Design of Mechanical Elements, John Wiley and Sons, Inc., New York, 1961.

Kilburn, R., "Reducing Wear in an Electromagnetic Clutch," Journal of Engineering for Industry, Trans. ASME, Series B., Vol. 90, No. 2, May, 1968.

Levy, H., "Temperature Distribution on Disc Clutches in Machine Tools," Ann. CIRP, Vol. 18, No. 2, May, 1970.

Libby, C. C., Motor Selection and Application, McGraw-Hill Book Company, Inc., New York, 1960.

Nelson, R. E., "Brake Linings as Related to Brake Drums and Brake Performance," SAE Paper, No. 670502, for meeting May 15-19, 1967.

Ramachandra, G., "Heat Generated During Clutch Engagement," RAO Institution of Engineers (India), Vol. 44, No. 7, pt. MEA, March, 1964.

Szoke, B., "On the Problem of Engaging and Disengaging Disc Clutches," Acta Technica (Budapest), Vol. 58, No. 3-4, 1967.

Town, H. C., "Design and Operation of Friction Clutches," Engineering Materials and Design, Vol. 8, No. 2, February, 1965.

Wrensch, B. E., "Applying Friction Clutches and Brakes," Electro-Technology, Vol. 79, No. 1, January, 1967.